On classification of Hadamard matrices of order 32

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IPM, September 2007

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(xxi) Definitive mathematics (e.g. a classification of all objects of a certain type; the final word on a mathematical topic)

Order	#Hadamard matrices
4	1
8	1
12	1
16	5
20	3
24	60
28	487
32	$\geq 3,578,006$

$$\begin{bmatrix} j_a & j_a & j_a & j_a \\ j_b & j_b & j_b & -j_b \\ j_b & j_b & -j_b & j_b \\ j_a & j_a & -j_a & -j_a \\ j_b & -j_b & j_b & j_b \\ j_a & -j_a & j_a & -j_a \\ j_a & -j_a & -j_a & j_a \\ j_b & -j_b & -j_b & -j_b \end{bmatrix}$$

$$\begin{bmatrix} ja & ja & ja & ja \\ jb & jb & jb & -jb \\ jb & jb & -jb & jb \\ ja & ja & -ja & -ja \\ jb & -jb & jb & jb \\ ja & -ja & ja & -ja \\ ja & -ja & -ja & ja \\ jb & -jb & -jb & -jb \end{bmatrix}$$

$a + b = n/4, \;\; a \geq b.$

$$\begin{bmatrix} j_a & j_a & j_a & j_a \\ j_b & j_b & j_b & -j_b \\ j_b & j_b & -j_b & j_b \\ j_a & j_a & -j_a & -j_a \\ j_b & -j_b & j_b & j_b \\ j_a & -j_a & j_a & -j_a \\ j_a & -j_a & -j_a & j_a \\ j_b & -j_b & -j_b & -j_b \end{bmatrix}$$

$a + b = n/4,\ a \geq b.$

This is of type b, where $0 \leq b \leq \lfloor n/8 \rfloor$.

A Hadamard matrix is of type b if

it has a set of four columns of type b and no set of four columns of type less than b.

Objective

To classify all Hadamard matrices of order 32 which are of type zero.

$$\begin{bmatrix} jm & jm & jm & jm \\ jm & jm & -jm & -jm \\ jm & -jm & jm & -jm \\ jm & -jm & -jm & jm \end{bmatrix}$$

$m = n/4$

Classification of combinatorial objects

A classification algorithm in general has two essential parts:

- Generation of objects
- Isomorphic rejection

The Read-Faradžev scheme

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An efficient procedure is needed to recognize those sub-objects which are not in canonical form.

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- The first three rows of H are in the following form:

$$\begin{bmatrix} e_m & e_m & e_m & e_m \\ e_m & e_m & -e_m & -e_m \\ e_m & -e_m & e_m & -e_m \end{bmatrix},$$

where $m = n/4$ and e_m denotes the all one row vector of dimension m.