IPM Logic Conference 2007,
June 10 - 15, 2007, IPM, Tehran, Iran

A Model Theoretic Characterization of $I\Delta_0 + Exp + B\Sigma_1$.

A. Enayat
American University
Washington, DC, USA

We establish the following model theoretic characterization of the fragment $I\Delta_0 + Exp + B\Sigma_1$ of Peano arithmetic in terms of *fixed points* of automorphisms of models of bounded arithmetic (the fragment $I\Delta_0$ of Peano arithmetic with induction limited to Δ_0-formulae).

Theorem A. *The following two conditions are equivalent for a countable model $\mathfrak{M}$ of the language of arithmetic:*

(a) $\mathfrak{M}$ *satisfies* $I\Delta_0 + B\Sigma_1 + Exp$.

(b) $\mathfrak{M} = I_{fix}(j)$ *for some nontrivial automorphism j of an end extension $\mathfrak{N}$ of $\mathfrak{M}$ that satisfies* $I\Delta_0$.

Here $I_{fix}(j)$ is the largest initial segment of the domain of j that is pointwise fixed by j, Exp is the axiom asserting the totality of the exponential function, and $B\Sigma_1$ is the Σ_1-collection scheme consisting of the universal closure of formulae of the form

$$[\forall x < a\ \exists y\ \varphi(x, y)] \rightarrow [\exists z\ \forall x < a\ \exists y < z\ \varphi(x, y)],$$

where φ is a Δ_0-formula. Theorem A was inspired by a theorem of Smoryński, but the method of proof of Theorem A is quite different and yields the following strengthening of Smoryński's result:

Theorem B. *Suppose $\mathfrak{M}$ is a countable recursively saturated model of PA and I is a proper initial segment of $\mathfrak{M}$ that is closed under exponentiation. There is a group embedding $j \longmapsto \widehat{j}$ from $Aut(\mathbb{Q})$ into $Aut(\mathfrak{M})$ such that $I = I_{fix}(\widehat{j})$ for every nontrivial $j \in Aut(\mathbb{Q})$. Moreover, if j is fixed point free, then the fixed point set of $\widehat{j}$ is isomorphic to $\mathfrak{M}$.*

Here $Aut(X)$ is the group of automorphisms of the structure X, and $\mathbb{Q}$ is the ordered set of rationals.