The 2nd One-day Seminar on Partial Differential Equation
March 13, 2008, IPM, Tehran

On the Existence of Periodic Solutions for Ordinary Differential Equations

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This lecture concerns the existence of periodic solutions for ODEs. There are some techniques for finding sufficient conditions for the existence of solutions for ODEs such as Fixed Point Theorems, Poincaré-Bendixon Theory, Massera Theorem, using the Poincaré map, etc. But the most common technique is the application of Fixed Point Theorems such as Banach, Brouwer, Schauder, Krasnoselskii, etc., and many mathematicians almost always use them. In this method, usually we need an appropriate mapping constructed by Green's function or the Poincaré map and a bound for solutions. However, you know that in many problems we do not have sufficient information about the boundary of the solutions and the Green's functions (and it may be more difficult). So, we cannot use these theorems in many problems.

On the other hand, from the functional analysis we know that, there are great tools for finding a solution for an operator equation. For example, some continuation theorems which can be found in Nonlinear Differential Equations present techniques for finding criteria for the existence of at least one solution for the operator equation $Lx = Nx$, where L and N are two operators with some special conditions. In this lecture, we try to apply these theorems for our problems.

In this lecture, some celebrated ODEs raised in Biology such as predator-prey systems are emphasized. The dynamical relationship between predators and their prey has long been one of the dominant themes in both ecology and mathematics, in part due to its universal existence and importance. At first sight, these problems may appear to be mathematically simple. However, they are very challenging and complicated. In fact, they are open problems from 1925.