Approximate stabilization of bilinear Schrödinger equations

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Due to the applications in the quantum control theory, the control of bilinear Schrödinger equations has been widely considered during the past decade. Many important results on the controllability, stabilization and optimal control of such systems have been achieved in finite dimensional cases. Note that, even though in many cases these finite dimensional systems may be seen as an approximation of an infinite dimensional one [4], it is not always true that the control properties of these approximations can be extended to the infinite dimensional system [6].

Concerning the infinite dimensional cases, the efforts on studying the controllability of such systems has been multiplied during the last few years. The general negative controllability result by [7] has been followed by some positive results on a test case corresponding to a particle in a moving infinite potential well [1, 2]. In a more generic configuration, Chambrion et al. [4] considered the approximate controllability of discrete-spectrum bilinear Schrödinger equations. In this result, the exact controllability properties of the finite dimensional Galerkin approximations of the system are extended to provide the approximate controllability of the main infinite dimensional one.

In this talk, contrarily to the previous works, we will first consider the case of Schrödinger equations with a mixed-spectrum. Even though, we will study the system starting at an initial state fully in the discrete part of the spectrum, the possibility of passage through the continuum, while being controlled, introduces new complications, namely the dispersion and the mass-lost phenomenon at infinity. We will see that through an appropriate Lyapunov technique, we may prevent these complications and ensure the approximate stabilization of any arbitrary equilibrium state in the discrete part of the spectrum [5].

Next, we will see that, while considering the stabilization of a discrete-spectrum Schrödinger equation, such kind of complications may also take place and can be seen as a mass-lost phenomenon at high-energy bound states. Then, the Lyapunov technique of the previous mixed-spectrum case can be adapted to ensure the approximate stabilization of this discrete-spectrum case under more generic assumptions [3].

References

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