به نام خداوند مهربان

Low-Density Parity-Check Codes Construction and Combinatorial Designs

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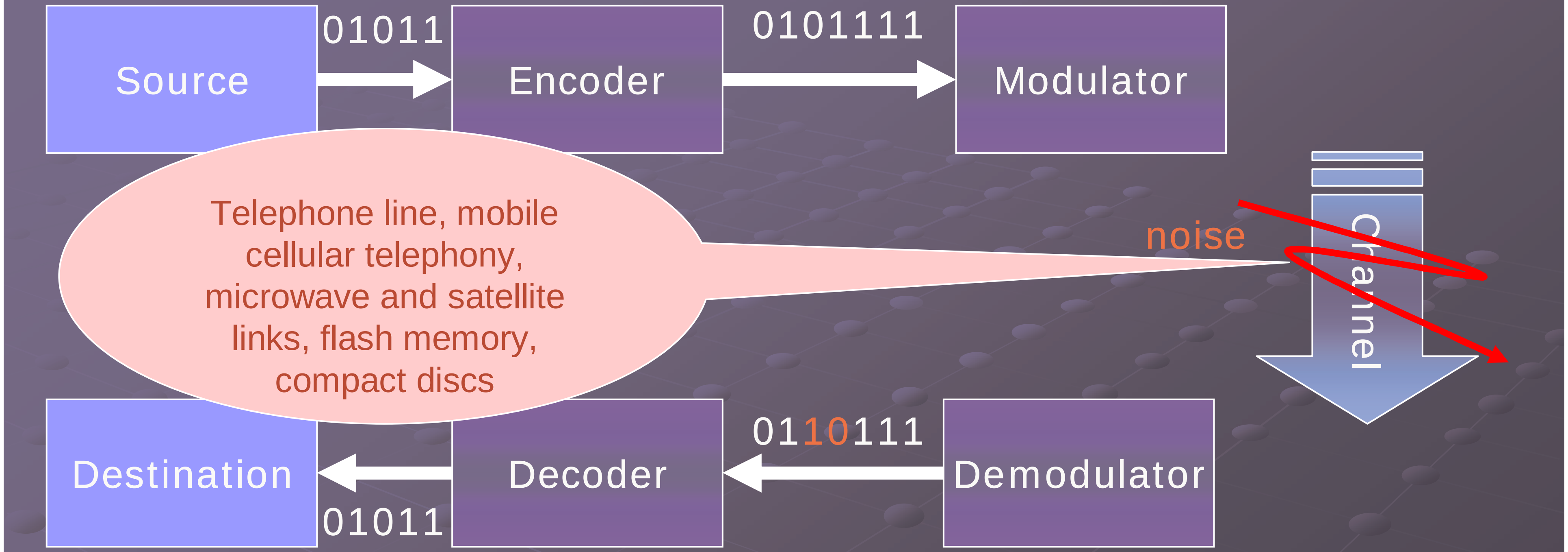
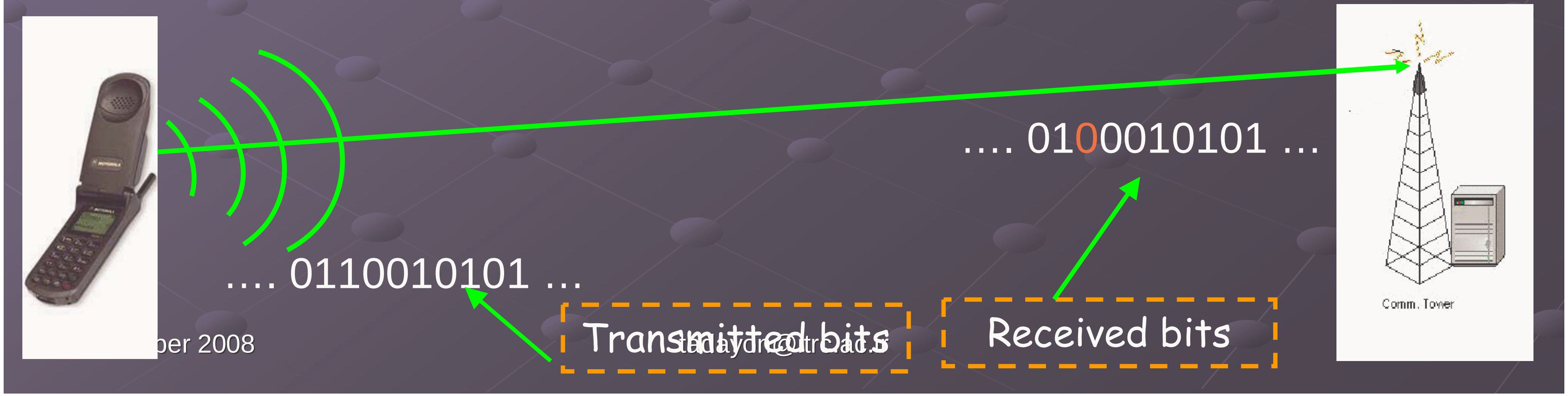
Outline

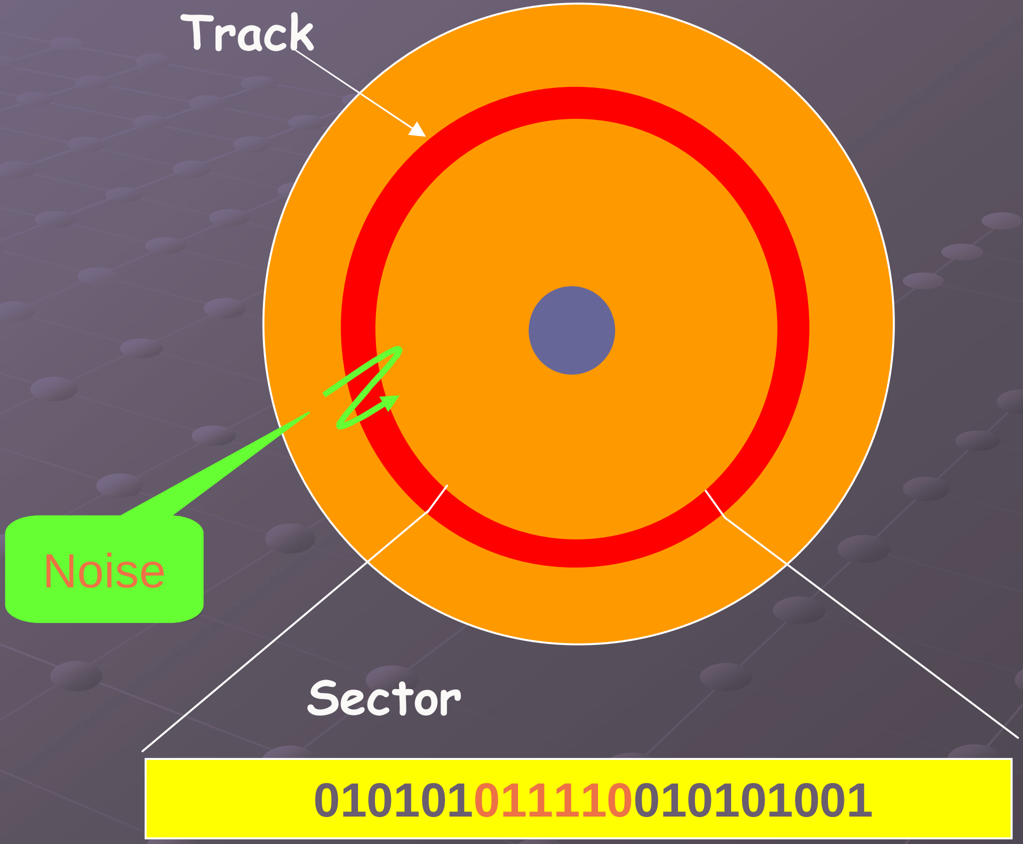
- Block Codes
- Noisy Channel Coding Theorem (Shannon Theorem)
- Low-Density Parity-Check (LDPC) Codes
- Combinatorial Designs and LDPC Codes
- Some properties

A Communication model


Source
01011
Encoder
0101111
Modulator
Channel
noise
Telephone line, mobile cellular telephony, microwave and satellite links, flash memory, compact discs
Demodulator
0110111
Decoder
01011
Destination



Cont.



Track

Noise

Sector

010101011111001010100 1

Block Codes

- [n, k]-code:

  A k dimensional subspace of F_q^n

- M=n-k= **Redundancy**

Code Rate

$$\mathrm{R} = \text{Code rate} = \frac{\text{Dimension}}{\text{Code length}} = \frac{k}{n}$$

$$0 < R < 1$$

Generator & Parity-Check Matrix

- G: k×n **generator matrix, which** $\mathbf{c} = \mathbf{m}G \in F_q^n$

$$C : (m_1, m_2, \ldots, m_k) \mapsto (c_1, c_2, \ldots, c_n)$$

$$(c_1, c_2, \ldots, c_n) = (m_1, m_2, \ldots, m_k) \times \begin{pmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots\cdot & g_{2n} \\ . & . & . & . \\ . & . & . & . \\ g_{k1} & g_{k2} & \cdots & g_{kn} \end{pmatrix}$$

- H: (n-k)×n **parity-check matrix**, such that GH^T = **0** in F_q

Example : [7, 4]-Hamming code

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

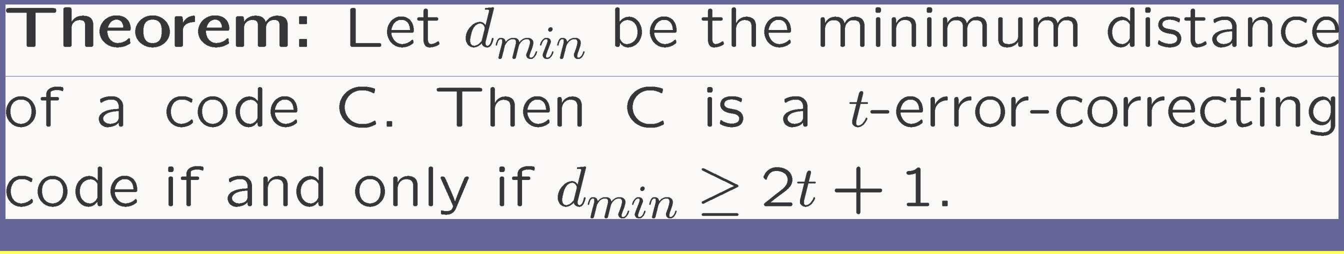
$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$y \in C$ iff $yH^T = \mathbf{0}$

$GH^T = \mathbf{0}$

Minimum Distance

- d: Minimum distance (the minimum weight of codewords)



Theorem: Let d_{min} be the minimum distance of a code C. Then C is a t-error-correcting code if and only if $d_{min} \geq 2t + 1$.

Higher minimum distance = Stronger code

Finding minimum distance:
NP hard

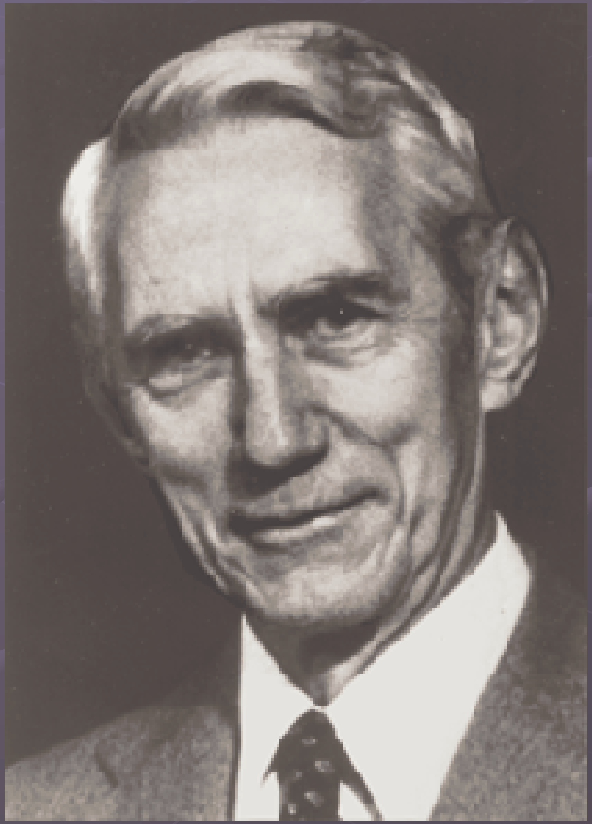
Linear Block Codes

- There are many practical linear block codes:
  - Hamming codes
  - Cyclic codes
  - Reed-Solomon codes
  - BCH codes
  - …

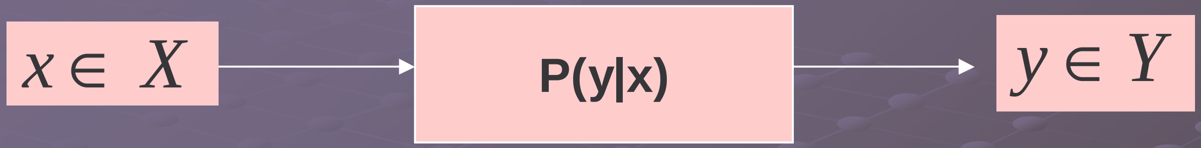
- But . . .

Shannon's Channel Coding Theorem

- In 1948, Claude Shannon published the paper: "A Mathematical Theory of Communications" which laid the foundations of Information Theory.



Noisy Channel Coding Theorem
(Shannon Theorem)



$$\text{Mutual Information } I(X;Y) = \sum_{x,y} P(x,y) \log \frac{P(x \mid y)}{P(x)}$$

$$\textit{Channel Capacity } C = \max_{P(x)} I(X;Y) \; {}^{bits}\!\!\diagup_{channel\ use}$$

$$\forall \;\; \varepsilon, \delta > 0, \; R : 0 < C - R < \varepsilon,$$

$$\text{a large length } n \text{ code of rate } R \text{ with } P_e < \delta$$

Error Control Coding

- good codes
  - ➢ Low-complexity encoding and decoding
  - ➢ Can approach channel capacity with low probability of error decoding

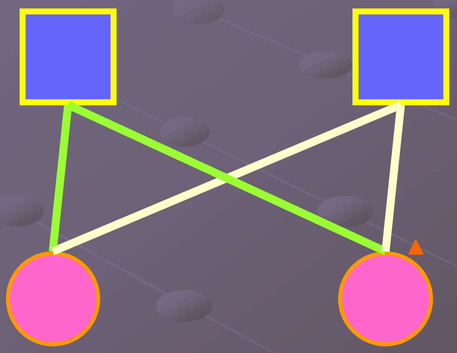
Low-Density Parity-Check (LDPC) Codes

- Gallager 1963, Tanner 1984, MacKay 1996
  - Linear block codes with sparse (small fraction of ones) parity-check matrices
  - Have simple representation in terms of bipartite graphs
  - Simple and efficient iterative decoding in the form of belief propagation
  - A class of channel capacity (Shannon limit) approaching codes

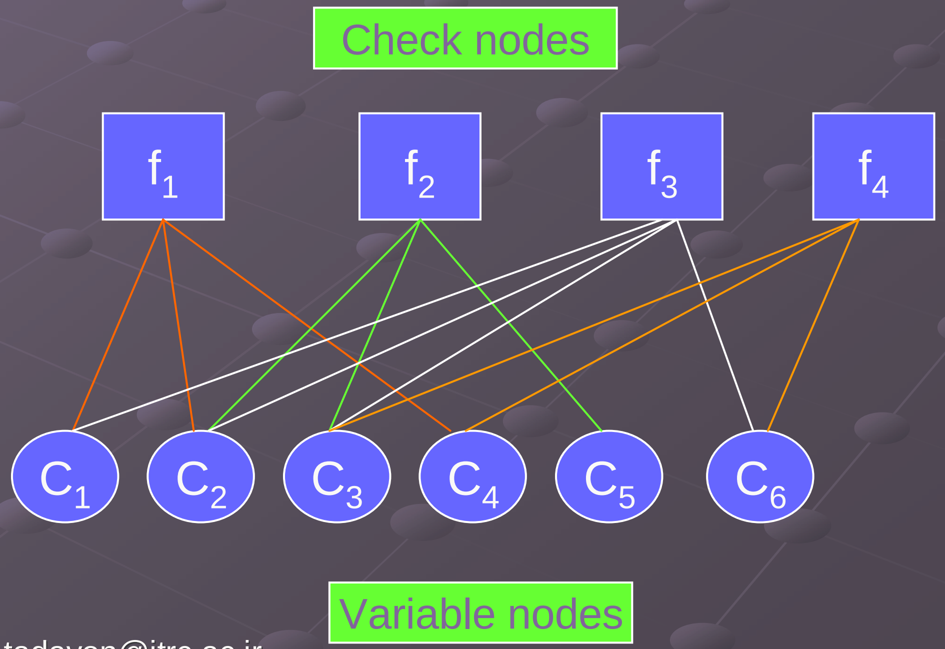
Graphical Representation

Example :

$$H=\begin{pmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \Rightarrow \mathbf{cH^T=0} \Rightarrow \begin{matrix} c_1+c_2+c_4=0 \\ c_2+c_3+c_5=0 \\ c_1+c_2+c_3+c_6=0 \\ c_3+c_4+c_6=0 \end{matrix}$$



cycle of length 4



LDPC codes construction types

- Random-like codes
  - Gallager (Low-density parity-check codes)
  - Mackay (Near Shannon limit performance of LDPC codes)
  - . . .
- Structured codes
  - Shu lin and . . . (Low-density parity-check codes based on finite geometries . . .)
  - S. Johnson and . . . (Codes for iterative decoding from partial geometries)
  - Fossorier (Quasi-cyclic low-density parity-check codes . . .)
  - Vasic and . . . (Combinatorial constructions of LDPC codes. . .)
  - Honary and . . . (Construction of LDPC codes based on BIBDs)
  - . . .

Complexities Comparison

Code	Encoding	Decoding
Random Linear Code	O(n^2)	$O(2^n)$

LDPC
Quasi-cyclic LDPC
	O(n^2)
	O(n)
		O(n)
		(Sum-Product Algorithm)

Decoding of linear codes:
NP hard

Gallager Codes

$$H = \begin{bmatrix} H_1 \\ H_2 \\ \vdots \\ H_{\omega_c} \end{bmatrix}_{\mu w_c \times \mu w_r}$$

Where for i=1,2,…,μ, the i-th row of submatrix H_1 contains of w_r 1’s in columns (i-1)w_r +1 to iw_r and other submatrices are simply column permutation of H_1.

Example: w_c=3, w_r=4

W_r=4

$$H = \left[\begin{array}{cccccccccccccccccccc}
1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1\\
\hline
1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0\\
0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&0\\
0&0&1&0&0&0&1&0&0&0&0&0&0&1&0&0&0&1&0&0\\
0&0&0&1&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1\\
\hline
1&0&0&0&0&1&0&0&0&0&0&1&0&0&0&0&0&1&0&0\\
0&1&0&0&0&0&1&0&0&0&1&0&0&0&0&1&0&0&0&0\\
0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&1&0\\
0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&1&0&0&0\\
0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1
\end{array}\right]$$

W_c=3

Drawback

- length-4 cycles in H
- encoding complexity

Mackay Codes

- H is created randomly
  - generating weight-w_c columns and (as near as possible) uniform row weight
  - generating weight-w_c columns, while ensuring weight-w_r rows, and no two columns having overlap greater than one(avoid cycle of length 4)

- Drawback: lack sufficient structure to enable low-complexity encoding

Example: $w_c=3$, $n=20, n-k=10$

$$H=\begin{pmatrix}
0&1&0&0&0&0&0&1&1&0&0&0&0&0&1&1&1&1&0&0\\
0&1&0&0&1&0&0&1&1&0&0&0&1&0&0&1&0&0&0&1\\
1&0&1&0&0&1&0&0&0&1&0&1&0&1&0&0&0&1&0&1\\
0&0&0&1&0&1&0&0&0&0&1&0&0&0&1&0&0&1&0&0\\
0&0&1&1&0&1&0&0&1&0&0&0&0&0&0&0&0&0&1&0\\
1&1&0&0&0&0&1&0&0&1&1&1&0&0&0&0&0&0&0&1\\
1&0&0&1&1&0&1&0&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&1&0&1&0&0&0&0&1&0&0&1&1&1&0&0&0&0&0\\
0&0&0&0&0&0&1&1&0&0&0&1&1&0&1&0&0&0&1&0\\
0&0&0&0&0&0&0&0&0&0&1&0&0&1&0&1&1&0&1&0
\end{pmatrix}$$

$w_r=7$

as near as possible uniform row weight

$w_r=6$

$w_r=5$

$w_c=3$

Combinatorial Designs and LDPC Codes

STS (v) & LDPC
Mackay and S. Johnson Codes

Example: STS(7) or 2- (7, 3, 1)

$$X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7\}$$

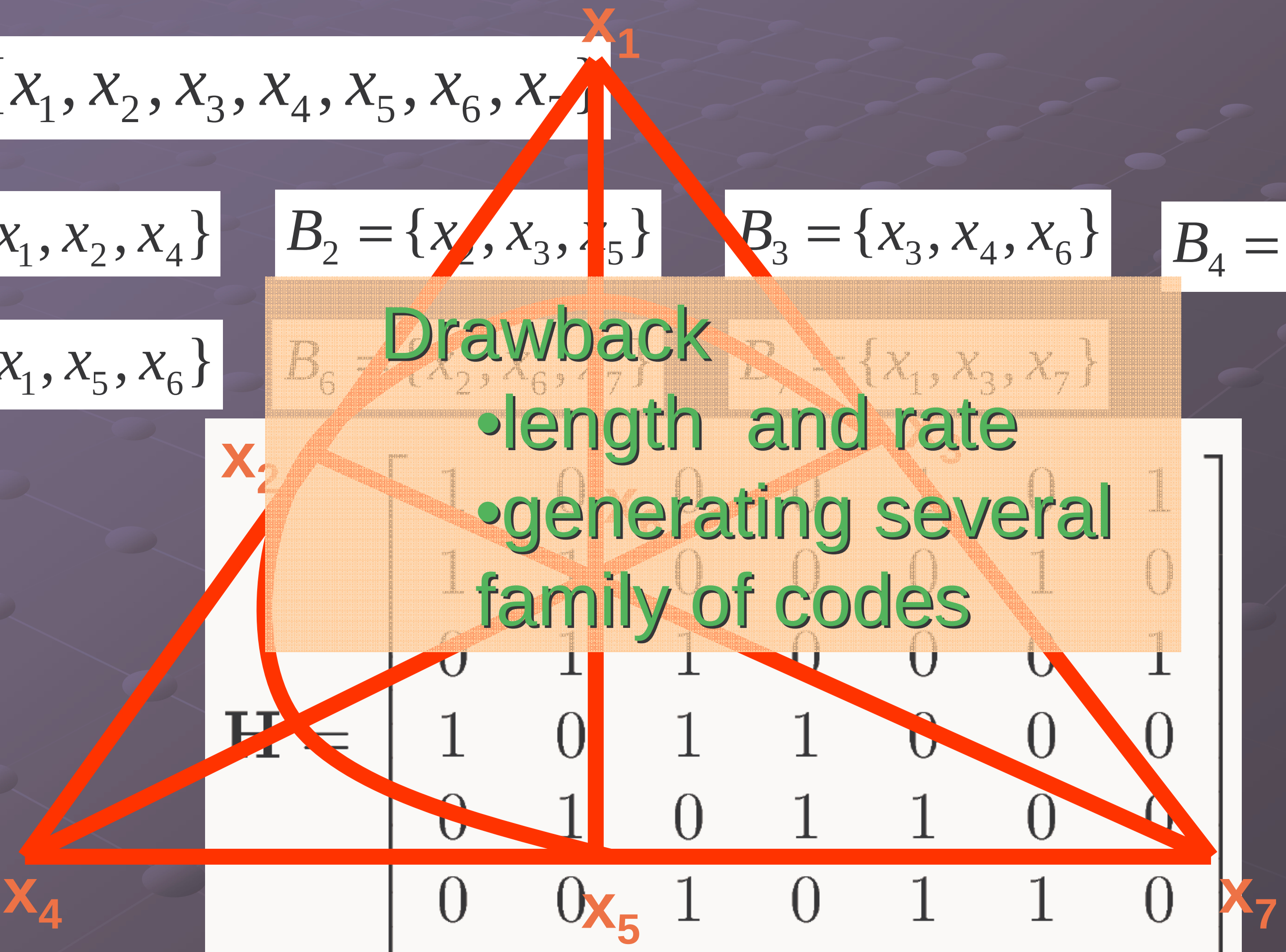
$B_1 = \{x_1, x_2, x_4\}$ $B_2 = \{x_2, x_3, x_5\}$ $B_3 = \{x_3, x_4, x_6\}$ $B_4 = \{x_4, x_5, x_7\}$

$B_5 = \{x_1, x_5, x_6\}$ $B_6 = \{x_2, x_6, x_7\}$ $B_7 = \{x_1, x_3, x_7\}$

$$\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Drawback

- length and rate
- generating several family of codes



Euclidian Geometries & LDPC

Shu lin and . . .

- Let *p* be a prime. Given two integers m≥2 and s≥1, the m-dimensional Euclidian geometry *EG(m, p^s)* over *GF(p^s)* consits of points, lines and hyperplanes (µ-flats).
  - A µ-flat is a µ-dimentional, 0≤µ≤m, subspace of the points of *EG(m, p^s)* over *GF(p^s)* .
  - A µ-flat has $p^{\mu s}$ points.
  - A point is a 0-flat and a line is a 1-flat.
  - A line contains p^s points
  - In *EG(m, p^s)* there are $p^{(m-1)s}(p^{ms}-1)/(p^s-1)$ lines and every point is the intersection of $(p^{ms}-1)/(p^s-1)$ lines
  - The set of points and lines of *EG(m, p^s)* form a 2-(p^{ms}, p^s, 1) BIBD

Example:

- Consider the 2-dimentional Euclidean geometry EG(2, 2^2). Let α be a primitive element of $F_{2^{2\times 2}}$. The incident vector for the line $L=\{\alpha^7,\alpha^8,\alpha^{10},\alpha^{14}\}$ is (0 0 0 0 0 0 0 1 1 0 1 0 0 0 1). The vector and its 14 cyclic shifts form the parity check matrix H.

$$H=\begin{pmatrix}
0&0&0&0&0&0&0&1&1&0&1&0&0&0&1\\
1&0&0&0&0&0&0&0&1&1&0&1&0&0&0\\
0&1&0&0&0&0&0&0&0&1&1&0&1&0&0\\
0&0&1&0&0&0&0&0&0&0&1&1&0&1&0\\
0&0&0&1&0&0&0&0&0&0&0&1&1&0&1\\
1&0&0&0&1&0&0&0&0&0&0&0&1&1&0\\
0&1&0&0&0&1&0&0&0&0&0&0&0&1&1\\
1&0&1&0&0&0&1&0&0&0&0&0&0&0&1\\
1&1&0&1&0&0&0&1&0&0&0&0&0&0&0\\
0&1&1&0&1&0&0&0&1&0&0&0&0&0&0\\
0&0&1&1&0&1&0&0&0&1&0&0&0&0&0\\
0&0&0&1&1&0&1&0&0&0&1&0&0&0&0\\
0&0&0&0&1&1&0&1&0&0&0&1&0&0&0\\
0&0&0&0&0&1&1&0&1&0&0&0&1&0&0\\
0&0&0&0&0&0&1&1&0&1&0&0&0&1&0
\end{pmatrix}$$

Cyclic LDPC
Low-complexity
encoding

Cycle shifts
of
The first row

Affine planes & LDPC
Shu lin and . . .

- Any 2-(n^2, n, 1) design is called an affine plane of order n, denoted Aff(n).
  - One line contains n points;
  - One point belongs to exactly n+1 lines;
  - Every line contains n points;
  - There are exactly n^2 points in Aff(n);
  - There are exactly n^2+n lines in Aff(n);
- For any prime power q there exists an affine plane of order q.

Example: Aff(n=3)

$$\mathbf{H}_{Aff} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}_{9\times 12}$$

Projective planes
Shu lin and . . .

- A 2-(n^2+n+1, n+1, 1) design is called a projective plane of order n, denoted by Pr(n).
  - One line contains n+1 points;
  - One point belongs to exactly n+1 lines;
  - Every line contains n+1 points;
  - There are exactly n^2+n+1 points in Pr(n);
  - There are exactly n^2+n+1 lines in Pr(n);

Example: Pr(n=3).

$$H_{\mathrm{Pr}oj} = \begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1
\end{bmatrix}_{13\times 13}$$

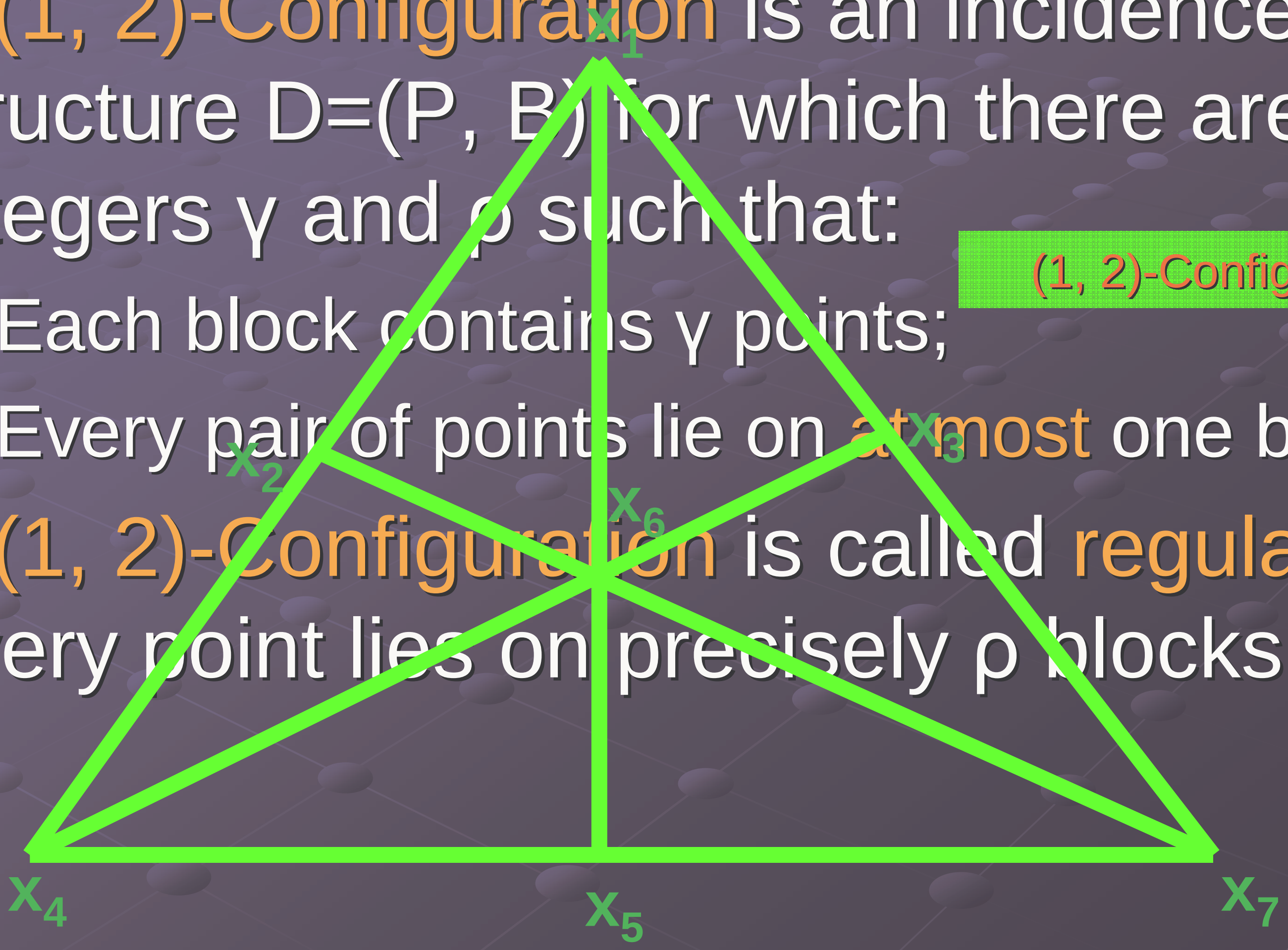
Quasi Cyclic LDPC codes

- A m×● ◡✓▲✕•⟍□□□•●□◂••⎾•□⎾⬍⎾✕⟍□ ⎾●▲✕⟍□•?□✓□⁻×⁻□□⎾•✕⎾▾◠✓●▲ ◡✓▲✕•⟍□ ⫽✕□□⎾✕ ⫽□◡✓▲✕•⟍□•●▲✕ ⫽⏌▾⎾⎾?□✓□□□□ ⎾ ⫽⏌⎾□•●⬍✓✕•✓●▲□▾●⏌⎾✕□▲•⎾□?••⏌▲□ ⫽⫽⎾✕✓▲• ⫽●□✓⟍□◡□⫽⫽?•▲•⫽●?□□
- □⟍□□□✕✕✓⟍□□□□□□□⎾ ⫽⏌⎾?□□□✕✕✓⟍□□γ□□⁻□□ ⁻□•?□✓ ◠✓●▲ ⫽⎾✕◡▾

$$\mathbf{H}_A = \begin{bmatrix} \mathbf{I} & \mathbf{I} & \cdots & \mathbf{I} \\ \mathbf{I} & \mathbf{P} & \cdots & \mathbf{P}^{q-1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{I} & \mathbf{P}^{\gamma-1} & \cdots & \mathbf{P}^{(\gamma-1)(q-1)} \end{bmatrix}$$

(1, 2)-Configuration & LDPC

- A (1, 2)-Configuration is an incidence structure D=(P, B) for which there are two integers γ and ρ such that:
  - Each block contains γ points;
  - Every pair of points lie on at most one block
- A (1, 2)-Configuration is called regular if every point lies on precisely ρ blocks.



(1, 2)-Configuration

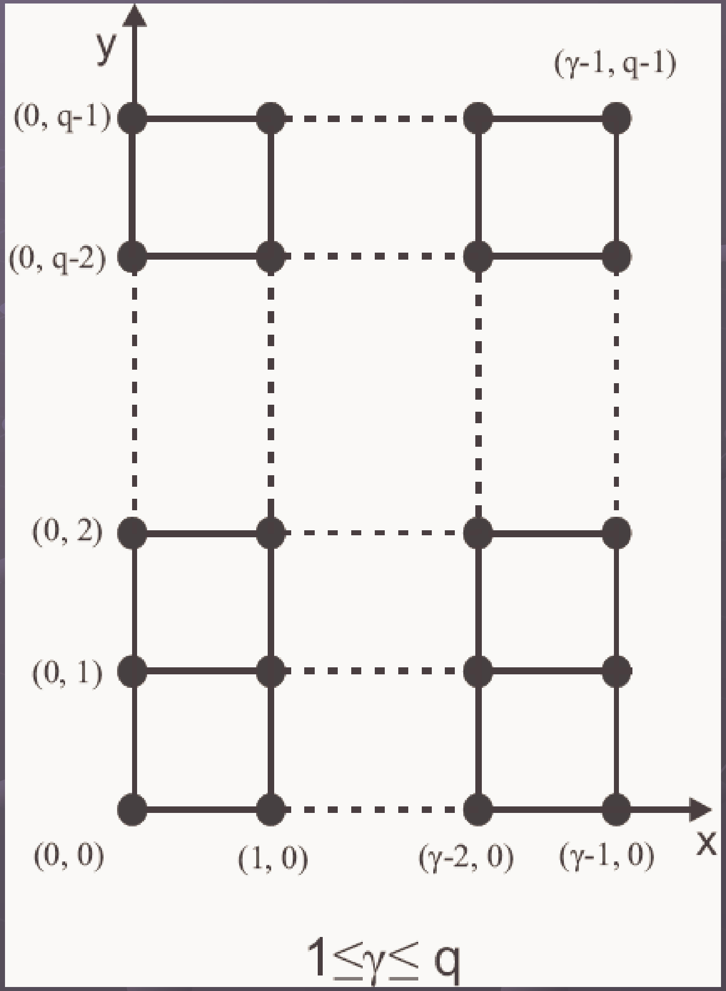
Why (1, 2)-configuration?

- Constructed LDPC codes have girth at least 6.
- Minimum distance of constructed LDPC codes is at least $\gamma+1$.
- Constructed LDPC codes have a good structure and can be represented as a cyclic or quasi-cyclic code

Integer Lattices
Vasic and . . .

- Integer lattices:

L(γ×q)={(x, y): 0≤ x ≤ γ-1, 0≤ y ≤ q-1}, γ≤q and γ, q are nonnegative integers.



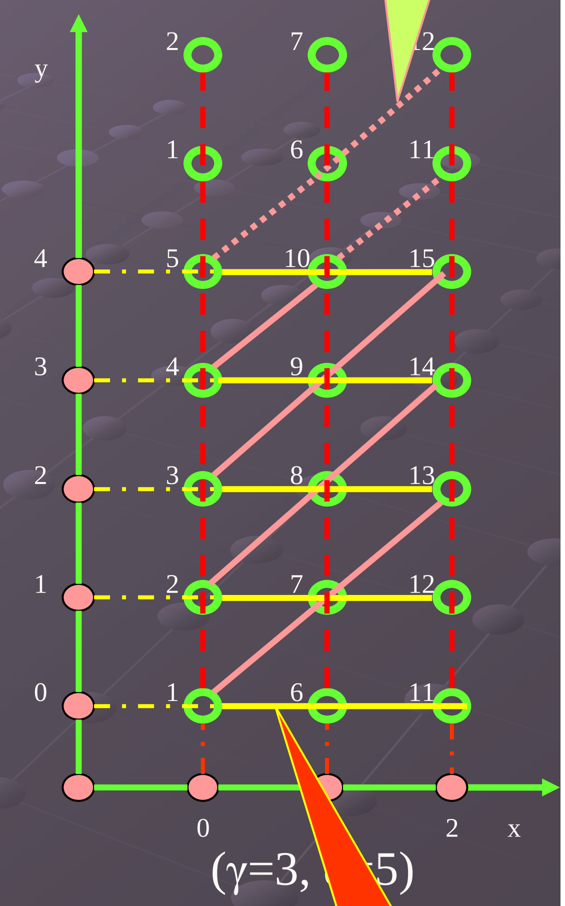
Integer Lattice designs

- The line $\ell_s(a)$ with slope $0 \leq s \leq q-1$ and passing through the point (0, a) is defined by:

$\ell_s(a)=\{(x, sx+a \pmod q),): 0 \leq x \leq \gamma-1\}$

L(γ, q)

- vq points
- q^2 lines
- any line contains γ points
- each point is in the intersection of q lines
- (1, 2)-configuration



Lines with slope s=1

Lines with slope s=0

Matrix representation

- Example: Lattice L(3,5)≡ Array (γ=3, q=5)

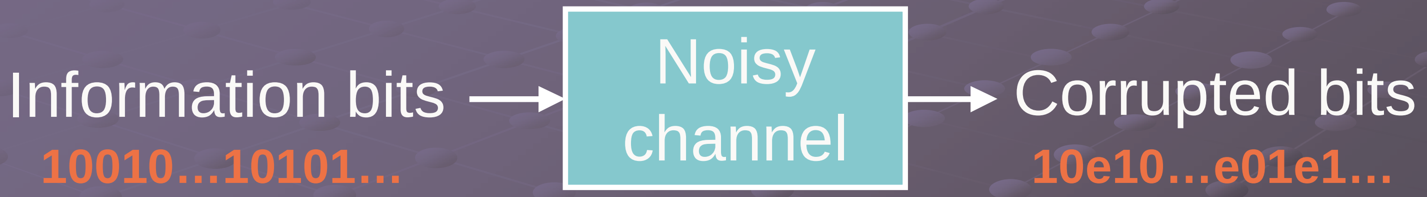
$$\mathbf{H}_{Latt} = \left[\begin{array}{ccccc|ccccc|ccccc|ccccc|ccccc}
1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0\\
0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0\\
0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0\\
0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0\\
0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1&0&0&0&0&1\\
\hline
1&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0\\
0&1&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&1&0&0&0&1&0&0\\
0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&1&0\\
0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0&0&0&0&0&1\\
0&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&1&0&0&0&0\\
\hline
1&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0\\
0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0\\
0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&0&0&0&0&0&1\\
0&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&1&0&0&0&0\\
0&0&0&0&1&0&0&1&0&0&1&0&0&0&0&0&0&0&1&0&0&1&0&0&0
\end{array}\right]_{15\times 25}$$

$$\mathbf{P} = \mathbf{P}^1 = \begin{bmatrix}
0&0&0&0&1\\
1&0&0&0&0\\
0&1&0&0&0\\
0&0&1&0&0\\
0&0&0&1&0
\end{bmatrix}.$$

QC-LDPC

$$\mathbf{H}_{Latt\,(3,5)} = \begin{bmatrix}
\mathbf{I}&\mathbf{I}&\mathbf{I}&\mathbf{I}&\mathbf{I}\\
\mathbf{I}&\mathbf{P}&\mathbf{P}^2&\mathbf{P}^3&\mathbf{P}^4\\
\mathbf{I}&\mathbf{P}^2&\mathbf{P}^4&\mathbf{P}^6&\mathbf{P}^8
\end{bmatrix}$$

Noisy Channels


Information bits

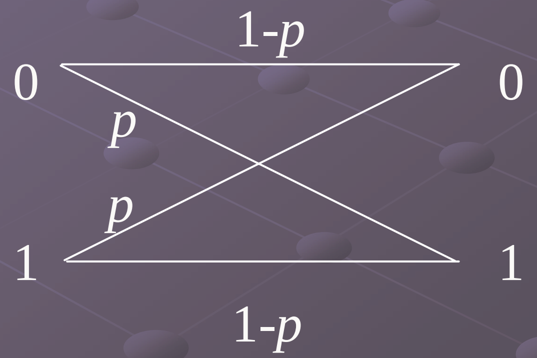
10010…10101…

Noisy channel

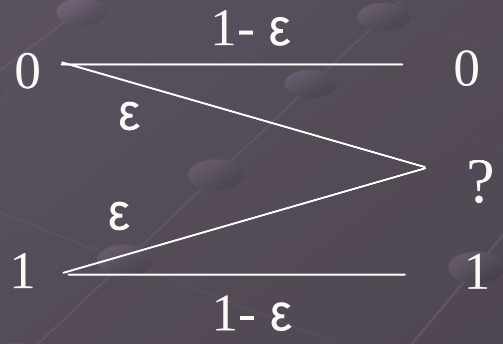
Corrupted bits

10e10…e01e1…

- Binary symmetric channel BSC (p)



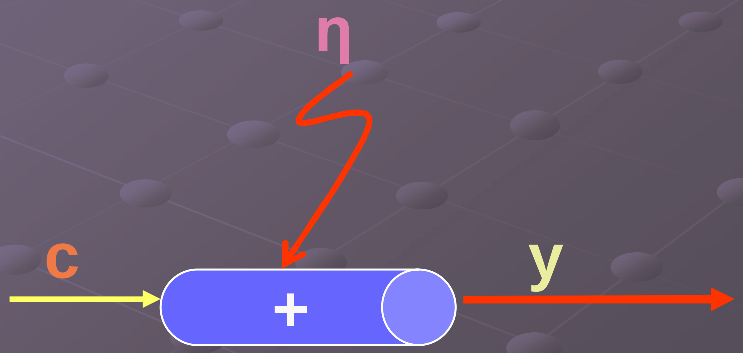
- Binary erasure channel BEC (ε)



AWGN (Additive White Gaussian Noise) channel

- c a codeword, η~ normal (μ=0, σ^2), y received word

$$y = c + \eta$$

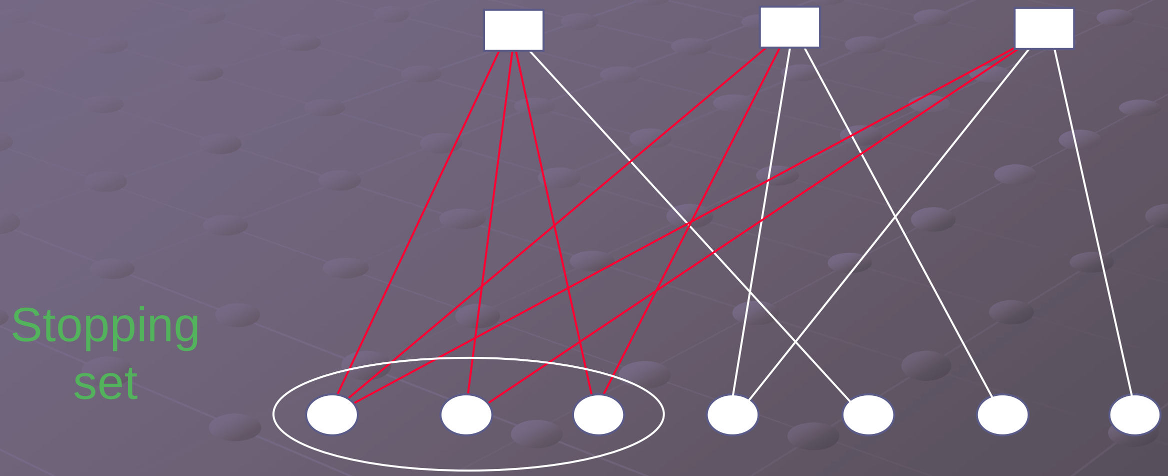


Stopping Sets
(a problem in binary erasure channels)

- A stopping set is a subset *S* of the variable nodes in Tanner graph of code C such that all the neighbors of S are connected to S at least twice.
- The size of the smallest non-empty stopping sets of code C is called the stopping distance

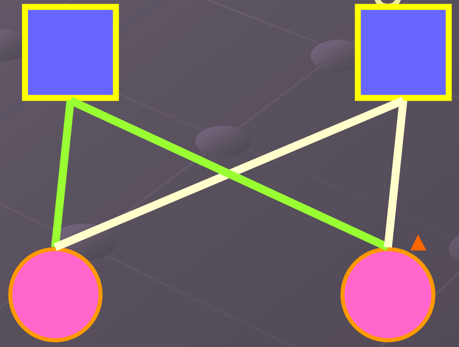
Finding stopping distance:
NP hard

Example: [7,4]-Hamming code



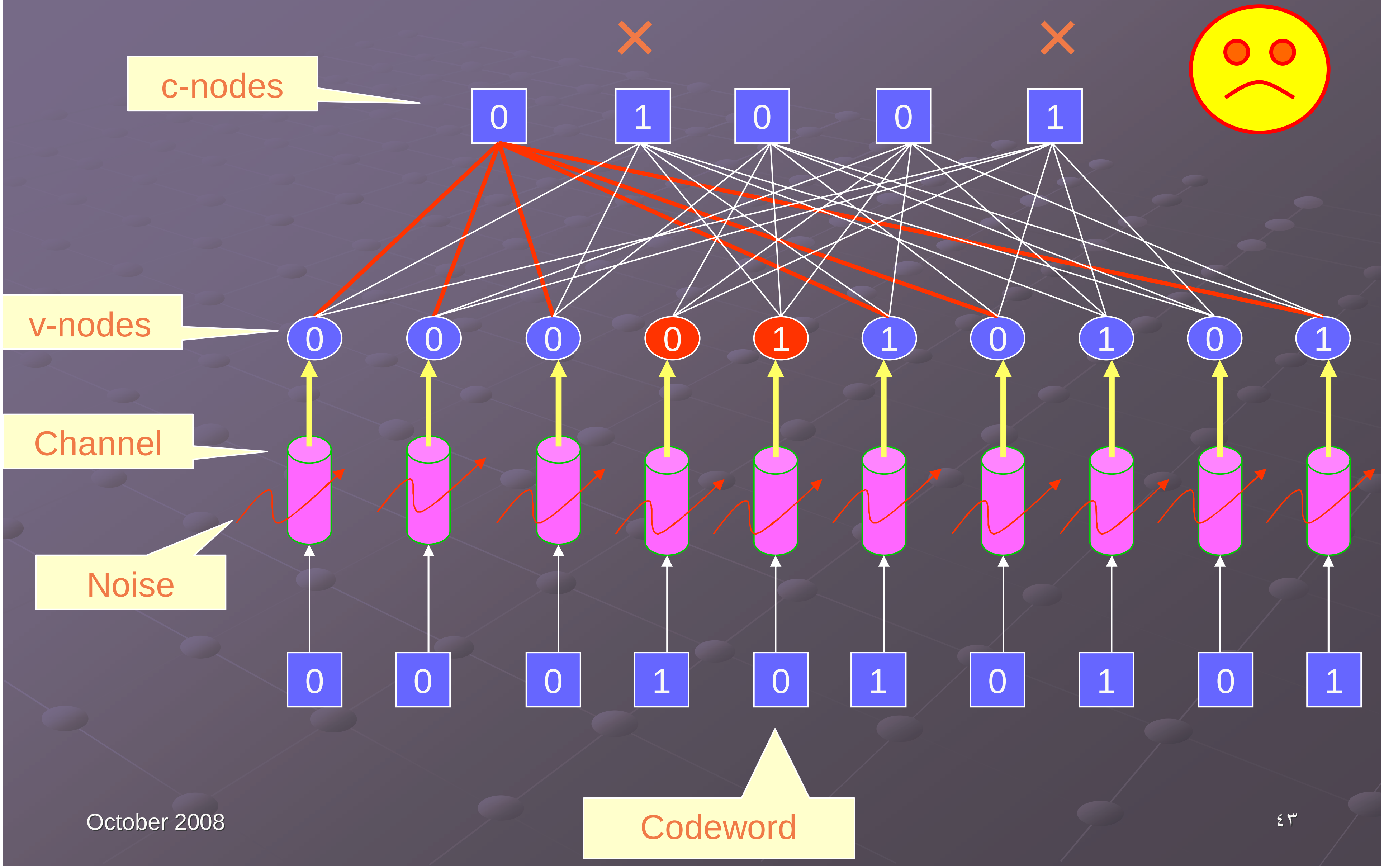
Iterative Decoding

- **Hard Decision** (passing of messages between variable nodes and check nodes are 0 or 1)
  - Bit-Flipping Algorithm (BFA)
- **Soft Decision** (passing of messages between variable nodes and check nodes are probabilistic)
  - Sum-Product Algorithm (SPA)
- Iterative decoding is optimal only if the code graph has no cycles
  - Want: to make girth (smallest cycle length) as large as possible;
  - Number of cycles of short length as small as possible;

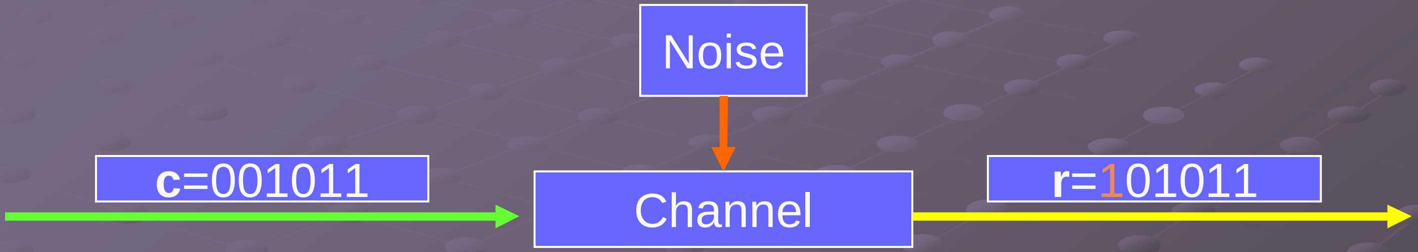


cycle of length 4

A graph representation of error detection



An example of Check and Variable nodes update (BFA)



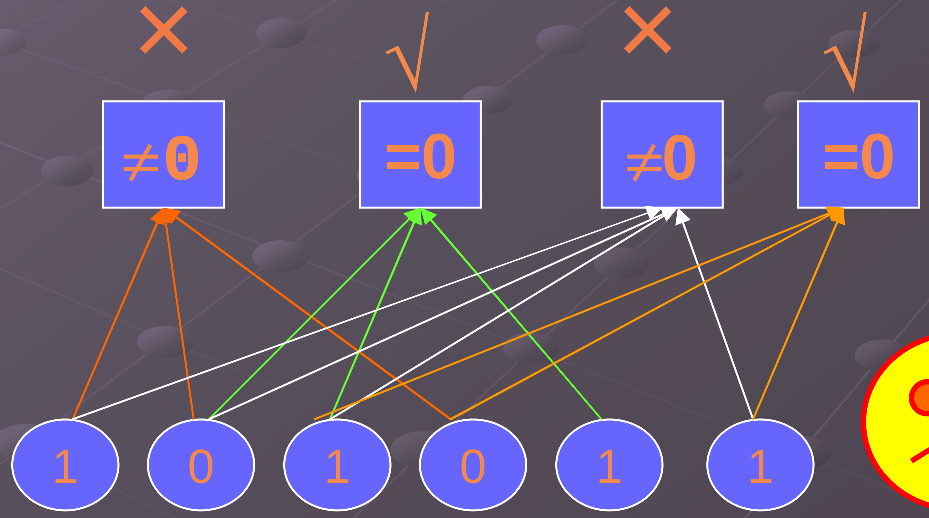
$cH^T=0$

$c_1+c_2+c_4=0$

$c_2+c_3+c_5=0$

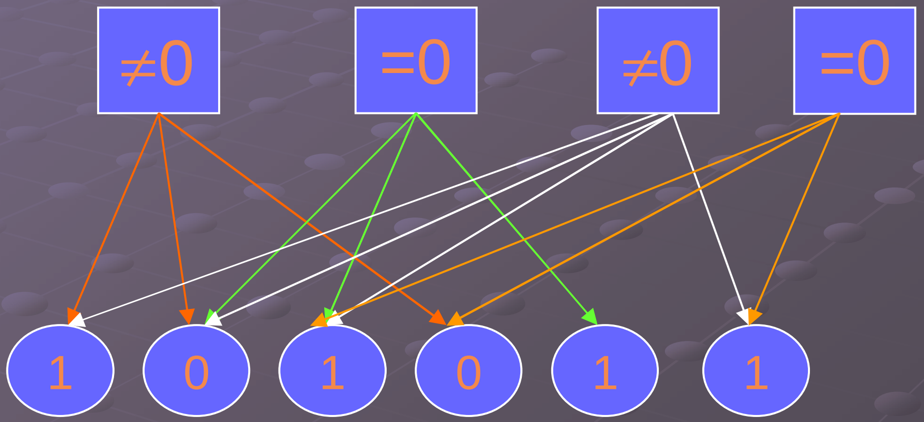
$c_1+c_2+c_3+c_6=0$

$c_3+c_4+c_6=0$

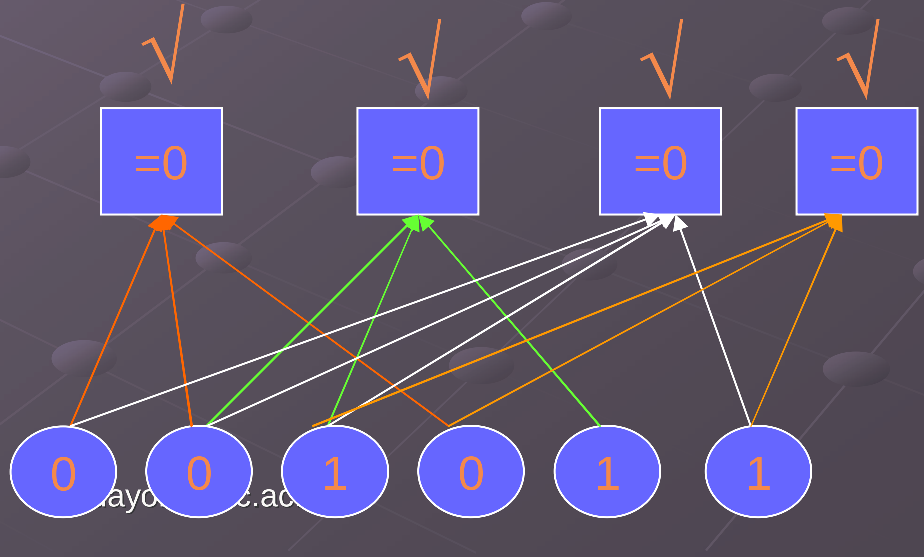


An example of Check and Variable nodes update (BFA)

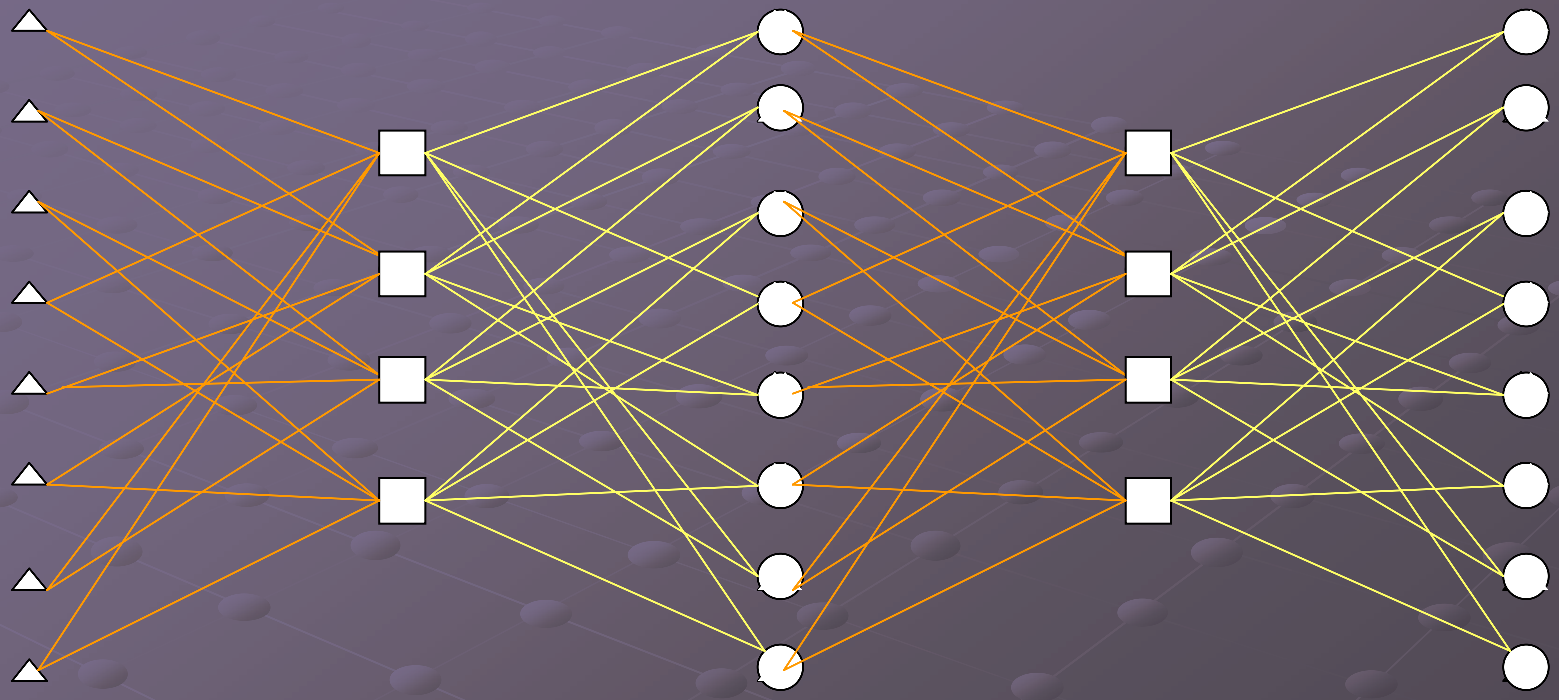
- Variable update



- Parity update



More Decoding



Posterior probabilities

checks

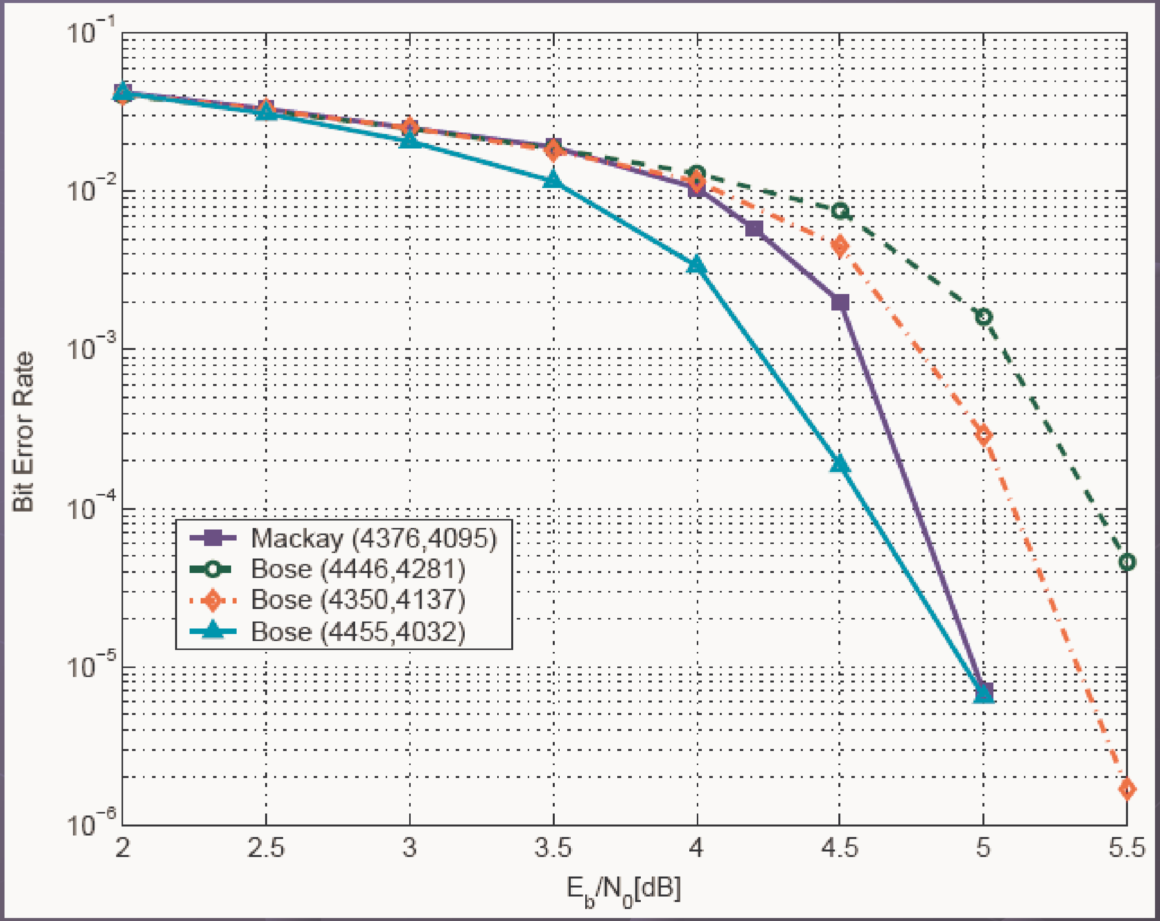
variables

checks

variables

Processing

Performance



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- O. Milenkovic, “On The Analysis And Application Of LDPC Codes”.
- A. Gomilko, “Turbo Codes overview”.
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Thank You