

Index theory, operator algebras and rho-invariants. Lectures in Teheran

Paolo Piazza

October 22, 2007

Abstract

The main topic of this series of lectures are ρ -invariants, their fundamental properties and their use in differential geometry and differential topology. Rho-invariants are secondary invariants attached to operators of Dirac type. In order to successfully investigate the properties of these secondary invariants it will be necessary to develop generalizations of the classic Atiyah-Patodi-Singer (\equiv APS) index theorem; we shall take this opportunity to talk, in general, about *higher* APS index theory.

1 First lecture: Atiyah-Patodi-Singer index theory on Galois coverings.

- The classic Atiyah-Patodi-Singer index theorem (repetition)
- Von Neumann invariants on Galois coverings
- Atiyah's index theorem on Galois coverings of closed manifolds
- L^2 -eta invariants
- The Atiyah-Patodi-Singer index theorem on Galois coverings (following Ramachandran)

2 Second lecture: the cylindrical approach to Atiyah-Patodi-Singer index theory

- b -calculus
- the APS index theorem via the b -calculus
- the APS index class
- the 0-degree higher APS index formula
- C^* -algebraic versus Von Neumann APS index theory
- Vaillant's proof of the Von Neumann APS index theorem
- The L^2 -signature formula on Galois coverings of manifolds with boundary

3 Third lecture: higher APS index theory

- Lott's approach to the Connes-Moscovici index theorem
- Noncommutative superconnections
- Higher eta invariants
- The higher APS-index theorem on Galois coverings
- Geometric applications.

4 Fourth lecture: bordism, rho-invariants and the Baum-Connes conjecture

- K-homology
- The Baum-Connes map for torsion-free discrete groups
- Consequences of surjectivity
- Consequences of injectivity
- Homotopy invariance of the signature rho-invariant under a Baum-Connes assumption
- Examples
- Vanishing of the spin rho-invariant in the presence of positive scalar curvature
- Higher rho-invariants

5 Fifth lecture: rho-invariants and determinants

- Eta invariants as (limits of) determinants of paths
- Keswani's approach to the homotopy invariance of $\rho(D^{\text{sign}})$ under a Baum-Connes assumption
 - loops and Bott's periodicity
 - consequences of surjectivity: the large-time path.
 - consequences of injectivity: the small-time path

6 Sixth lecture: groups with torsion, bordism and rho-invariants

- Bordism of metrics of positive scalar curvature
- Rho-invariants are bordism invariants
- Groups with torsion and nonbordant metrics
- Groups with torsion and a theorem of Chang-Weinberger