On the Energy of Graphs and Multigraphs

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Let G be a graph and $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of its adjacency matrix. The energy of G, denoted by $E(G)$ is defined as $\sum_{i=1}^{n} |\lambda_i|$. For any matrix $A \in M_n(\mathbb{C})$, we define the energy of A, $E(A) = \sum_{i=1}^{n} |\lambda_i|$, where $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A. We obtain some results on the effect of duplicating any edge of a graph on the energy of the graph. Specifically we show that if A, B are Hermitian matrices and ϵ is a positive real number such that $E(A - \epsilon B) < E(A)$, then for any two real numbers α, β with $\alpha > \beta \geq 0$, $E(A + \alpha B) > E(A + \beta B)$.

We also investigate the relations between the energy of a graph and their subgraphs. Day and So proved that if $H_1, \ldots, H_k$ are subgraphs of G such that the edges of $H_1, \ldots, H_k$ is a partition of the edges of G, then $E(G) \leq \sum_{i=1}^{k} E(H_i)$. We conjectured that if the edges of $H_1, \ldots, H_k$ covers the edges of G, then $E(G) \leq \sum_{i=1}^{k} E(H_i)$, and showed that this conjecture is true for forests and multipartite graphs.

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