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Spherical Designs and Association Schemes Versus Euclidean Designs and Coherent Configurations

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The concept of spherical t-design, which is a finite subset of the unit sphere, was introduced by Delsarte-Goethals-Seidel (1977). The concept of Euclidean t-design, which is a two step generalization of spherical design in the sense that it is a finite weighted subset of Euclidean space, by Neumaier-Seidel (1988). We first review these two concepts and examples, as well as the concept of tight t-design, i.e., the one whose cardinality reaches the natural lower bound. We are interested in t-designs (spherical or Euclidean) which are either tight or close to tight. As is well known by Delsarte-Goethals-Seidel (1977), in the study of spherical t- designs and in particular of those which are either tight or close to tight, association schemes play important roles. The main purpose of this talk is to show that in the study of Euclidean t-designs and in particular of those which are either tight or close to tight, coherent configurations play important roles. Here, coherent configuration is a purely combinatorial concept defined by D. G. Higman, and is obtained by axiomatizing the properties of general, not necessarily transitive, permutation groups, in the same way as association scheme was obtained by axiomatizing the properties of transitive permutation groups.

As an application of this general theory, we discuss the current status of our research to try to classify Euclidean 4-designs (X, w) on two concentric spheres $S = S_1 \cup S_2$ centered at the origin whose weight function is constant on each $X \cap S_i (i = 1, 2)$ and each $X \cap S_i (i = 1, 2)$ is at most 2-distance set. We will also give two new families of feasible parameters for coherent configurations, one family is obtained from Euclidean tight 4-designs on two concentric spheres and another family is obtained from non-tight Euclidean 4-designs.