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Intersection theorems for finite sets

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Let us consider a finite set X of n elements and a family $\mathcal{F}$ of distinct subsets of X. Extremal set theory deals with (mostly) upper bounds on the size of $\mathcal{F}$ subject to some conditions on the members of $\mathcal{F}$. Some of the most important theorems in this field deal with conditions related to the size of the intersection of members of $\mathcal{F}$. For example, for positive integers r and t (with r at least 2) a family $\mathcal{F}$ is called r-wise t-intersecting if any r members of $\mathcal{F}$ intersect in at least t elements. The by now classical theorem of Katona determines the maximal size of $\mathcal{F}$ for the case of $r = 2$. The general situation for r greater than 2 is still open. The obvious construction of considering the family of all supersets of a fixed subset of t elements has size 2^{n-t}. I proved that this is best possible if and only if t is at most $2^r - r - 1$. In this talk we consider these and related problems.