Chain Partitions of Normalized Matching Posets

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The subsets of a finite set, the divisors of a positive integer, and the subspaces of a finite dimensional vector space over a finite field give examples of normalized matching posets. A finite graded poset is a normalized matching poset (or has the LYM property) if, for any two consecutive levels $X$ and $Y$ and every subset $S \subseteq X$, we have

$$\frac{|\Gamma(S)|}{|Y|} \geq \frac{|S|}{|X|},$$

where $\Gamma(S)$ is the set of elements of $Y$ that are related to some element in $S$.

In what ways can we partition a normalized matching poset into linearly ordered subsets (chains)? We give a common generalization of three decades old conjectures (by Griggs and Füredi) and report on some recent progress.