A Survey of Alternating Permutations

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A permutation $a_1 a_2 \cdots a_n$ of $1, 2, \ldots, n$ is *alternating* if $a_1 > a_2 < a_3 > a_4 < \cdots$. We will survey the enumerative properties of alternating permutations. In particular, if E_n is the number of alternating permutations of $1, 2, \ldots, n$, then

$$\sum_{n \geq 0} E_n \frac{x^n}{n!} = \sec x + \tan x,$$

a famous result of Désiré André. The talk will also include some examples of where E_n occurs in other enumerative problems, the distribution of the longest alternating subsequence of a random permutation, some umbral formulas for enumerating classes of alternating permutations, and the special role of alternating permutations in the problem of counting permutations $a_1 a_2 \cdots a_n$ where we specify the set $\{i \colon a_i > a_{i+1}\}$.