

## Spreading blocks and subgraphs

**Richard M. Wilson**

*California Institute of Technology  
USA*

One of the ideas used in the construction of designs and decompositions of graphs on large numbers of points is that of "spreading blocks". One starts with a design, or a multi-decomposition of the edges of a complete graph, with a large index  $\lambda$  on  $v$  points. This is used as a basis for a construction of a group-divisible design, or a decomposition of a complete multipartite graph, with index 1 and where the number  $v'$  of points is a multiple of  $\lambda v$ . In practice,  $\lambda = q$  is a power of a prime, and  $v' = vq^d$  where  $d$  is large. The set of points is the cartesian product of the  $v$  points with a  $d$ -dimensional vector space over the field of  $q$  elements.

We survey some of the applications of this technique and give a proof of a variation of a theorem of John Blanchard: For any  $t$  and  $k$ ,  $t \leq k$ , and any positive integer  $n$ , there exist transversal designs with  $k$  groups of size  $nq^d$ , strength  $t$ , and index 1 for all sufficiently large primes  $q$  and integers  $d$ .