Exterior Algebras and Two Conjectures about Finite Abelian Groups

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In 1999, Hunter Snevily made the following conjecture.

Conjecture. Let G be an abelian group of odd order and let $A, B \subseteq G$ satisfy $|A| = |B| = k$. Then the elements of A and B may be ordered $A = \{a_1, a_2, \ldots, a_k\}$ and $B = \{b_1, b_2, \ldots, b_k\}$ so that the sums $a_1 + b_1, a_2 + b_2, \ldots, a_k + b_k$ are pairwise distinct.

The motivation for this question comes from the study of Latin squares. A *transversal* of a Latin square is a collection of cells, no two of which are in the same row or column, and we say that a transversal is *Latin* if no two of its cells contain the same symbol. Latin transversals are nice structures to find in Latin squares. The above conjecture can be rephrased in terms of Latin transversals as follows: Every $k \times k$ submatrix of the addition table of an abelian group of odd order has a Latin transversal.

There is a related conjecture due to Dasgupta, Karolyi, Serra, and Szegedy arising from an attempt to prove Snevily's conjecture. In this talk, we first survey some known results on these conjectures. Then we will discuss our recent work on these conjectures by using exterior algebras and characters of abelian groups.

(Joint work with Tao Feng and Zhi-Wei Sun)