Plenary talk: Large deviations for regularly varying random walks

Through the last twenty years the notion of regular variation has been one of the dominating concepts for modeling extremes. Early on, in his 1986 paper in Adv. Appl. Probab. and in his 1987 book, Sid Resnick propagated the use of this concept for extreme value purposes; see also his 2006 book "Heavy tail phenomena". More recently, following the Ann. Probab. 2001 paper by Lin and de Haan, Hult and Lindskog developed some kind of a calculus for regularly varying structures, see their paper in Stoch. Proc. Appl. in 2005. The latter approach allows one to consider functional regular variation in close relation with weak convergence, in particular they proved necessary and sufficient conditions for functional regular variation in the spirit of weak convergence (convergence of the finite dimensional distributions and tightness) and they also showed an analog of the continuous mapping theorem for regular variation.

The paper Hult, Lindskog, Mikosch, and Samorodnitsky "Functional large deviations for multivariate regularly varying random walks" Ann. Appl. Probab. 15, 2005, is the basis for this talk. In it, it is shown that the notion of functional regular variation in Skorokhod space extends in a natural way to so-called heavy-tailed large deviation principles. The first results on heavy-tailed large deviations for real-valued random variables go back to Linnik in the 1960s, and C. Heyde considered some special cases. A.V. Nagaev formulated the general principles of heavy-tailed large deviations for large classes of distributions. In a sense, these are extensions of the defining property of a subexponential distribution. In the above mentioned paper Hult et al., the A.V. Nagaev result was extended to the functional setting. In turn, it allows one to derive tail probabilities for functionals of a heavy-tailed random walk, such as the ruin probability.

Dependence and tail modeling with applications

I, Evidence and modeling of extremes in finance, insurance and telecommunications

The aim of this talk is to give some evidence of heavy tail phenomena in areas as diverse as insurance, finance, telecommunications. We discuss simple statistical tools for detecting heavy tails in a one-dimensional sequence. Then we consider some suitable distributions for the description of extremes. We give special attention to distributions with power law tails which have been playing a significant role in extreme value theory and the statistics of extremal events. We also consider some applied stochastic processes which are used for describing extremal behavior in the areas mentioned.

II. Extremes of financial time series

Financial time series are widely available. Various models have been fitted to financial data. Among those are the GARCH and the stochastic volatility (SV) models. Whereas the GARCH model prescribes a rather complicated relationship between the noise sequence and the volatility sequence, the SV models assumes independence between the two models. These structural differences lead to completely different extremal behavior which is reflected in the asymptotic behavior of the sample correlation functions and of the maxima and upper order statistics in a sample. Typically, SV models do not have clusters of extremes, in contrast to GARCH and related models. The extreme value theory (EVT) of SV models is rather simple; it parallels the one for an iid sequence. In contrast, the EVT of GARCH models depends on the theory of stochastic recurrence equations, going back to work by Kesten (1973) and Godlie (1991) about the tails of the solution to such equations.

III. The extremogram: a correlogram for extreme events

This talk is based on the paper of Davis and Mikosch in Bernoulli 2009 which bears the same name. We consider a strictly stationary process. There exist various well known measures of the strength of dependence in such a process. Among them, the autocorrelation function is most popular. The latter function cannot say much about the dependence of very large and very small values in the process. A quantity which, to some extent, captures some of the extremal dependence between the components of a bivariate vector is the tail dependence coefficient. Originally used by extreme value specialists such as Ledford and Tawn in the 1990s, the tail dependence coefficient became popular in quantitative risk management. The extremogram is a time lag-wise tail dependence function. It can be interpreted as the autocorrelation function of some second order stationary process. We discuss examples and estimation problems.