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Castelnuovo-Mumford Regularity And Linearity Defect

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Given a finitely generated module M over a commutative local ring (or a standard graded K-algebra) R the linearity defect of M, denoted by $ld_R(M)$, is an invariant that measures how far M and its syzygies are from having a linear resolution.

In this talk we are going to investigate relations between $ld_R(M)$ and the Castelnuovo-Mumford regularity of M, $reg(M)$. To this end, we use stable filtration of modules.

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Geometrical Methods in Local Algebra

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In this talk we try to understand the geometry of certain algebraic results, i.e., we review several algebraic results with purely geometric arguments.

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Cohomological Dimension Filtration and Annihilators of Top Local Cohomology Modules

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Iran

Let $\mathfrak{a}$ denote an ideal in a commutative Noetherian ring R and M a finitely generated R-module. In this talk, we introduce the concept of the cohomological dimension filtration $\mathscr{M} = \{M_i\}_{i=0}^{c}$, where $c = \operatorname{cd}(\mathfrak{a}, M)$ and M_i denotes the largest submodule of M such that $\operatorname{cd}(\mathfrak{a}, M_i) \leq i$. Some properties of this filtration are investigated. In particular, in the case that $(R, \mathfrak{m})$ is local and $c = \dim M$, we are able to determine the annihilator of the top local cohomology module $H^c_{\mathfrak{a}}(M)$. In fact, it is shown that $\operatorname{Ann}_R(H^c_{\mathfrak{a}}(M)) = \operatorname{Ann}_R(M/M_{c-1})$. As a consequence, it follows that there exists an ideal $\mathfrak{b}$ of R such that $\operatorname{Ann}_R(H^c_{\mathfrak{a}}(M)) = \operatorname{Ann}_R(M/H^0_{\mathfrak{b}}(M))$.

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Finiteness Conditions on the Set of Maximal Subrings of a Commutative Ring

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In this talk we are interested in showing that $RgMax(R)$ is infinite for certain commutative rings R, where $RgMax(R)$ is the set of all maximal subrings of R. We characterize certain commutative rings with only finitely many maximal subrings (up to isomorphism) and also find some equivalent finiteness conditions (such as chain conditions) for such rings.

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Virtual Gorensteinness and Induction from Elementary Abelian Groups

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Let Λ be an artin algebra. It is known [2] that, $\mathrm{Gp}(\Lambda)^{\perp} = {}^{\perp}\mathrm{Gi}(\Lambda)$, where $\mathrm{Gp}(\Lambda)$ and $\mathrm{Gi}(\Lambda)$ denote the subcategories of finitely generated Gorenstein projective and Gorenstein injective modules respectively and the symbol $^{\perp}$ refers to the Ext^1_{Λ}-orthogonal classes. However, there are examples of artin algebras for which $\mathrm{GP}(\Lambda)^{\perp} \neq {}^{\perp}\mathrm{GI}(\Lambda)$ where $\mathrm{GP}(\Lambda)$ and $\mathrm{GI}(\Lambda)$ are the subcategories of Gorenstein projective and Gorenstein injective modules; see [1]. Therefore, by definition, an artin algebra Λ is said to be *virtually Gorenstein*, provided $\mathrm{GP}(\Lambda)^{\perp} = {}^{\perp}\mathrm{GI}(\Lambda)$. The notion of virtually Gorenstein algebra was first introduced in [3], as a generalization of the notion of Gorenstein algebra, that is, an algebra with finite self-injective dimension from both sides, and studied extensively in [2] and [1]. It is worth pointing out that, the importance of virtually Gorenstein algebras stems from the fact that, besides that they provide a common generalization of Gorenstein algebras and algebras of finite representation type, in addition they form a rather well-behaved class of algebras for testing various homological conjectures. In fact, it is known that the Gorenstein Symmetry Conjecture holds for virtually Gorenstein algebras; see [2, Theorem 11.4]. Recently, it is shown by Beligiannis that an artin algebra of finite Cohen-Macaulay type is virtually Gorenstein if and only if every Gorenstein projective Λ-module is a direct sum of finitely generated modules.

According to the mentioned results, it is an interesting to study the property of virtually Gorenstein algebras in the context of group algebras. More precisely, the focus of this note lies in the problem of when the group algebra $\Lambda\Gamma$, where Γ is a finite group and Λ is an artin algebra, is virtually Gorenstein. Our main result in this direction asserts that, $\Lambda\Gamma$ is a virtually Gorenstein algebra if and only if $\Lambda\Gamma'$ is virtually Gorenstein, where Γ' runs over all elementary abelian subgroups of Γ. This is in agreement with several classical results for testing several homological or representation-theoretic properties, for instance projectivity, in group rings of finite groups; see [5] and [4]. We also prove that, under mild assumptions, being virtually Gorenstein algebra carries over from a group Γ to its subgroups and vice versa. These results based on a joint work with Sh. Salarian.

Keywords: Gorenstein projective modules, virtually Gorenstein algebra, Group algebras.

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Cofiniteness, Abelian Categories and Local Cohomology Modules

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Iran

In this lecture we consider the following Hartshorne's question, given in [4]:
For a given ideal I of a Noetherian ring R, is the category $\mathfrak{M}(R,I)_{cof}$ of I-cofinite modules forms an Abelian subcategory of the category of all R-modules? That is, if $f : M \longrightarrow N$ is an R-homomorphism of I-cofinite modules, are $\ker f$ and $\operatorname{coker} f$ I-cofinite?
Then we present a family of results proved by several authors which answer this question affirmatively. Kawasaki conjectured that for the special case $\dim(R/I) = 1$ this wild problem has an affirmative answer in general (See [5, p.125]). Recently, Bahmanpour, Naghipour and Sedghi in [2], have presented an affirmative answer to Kawasaki's conjecture.
In this lecture we talk about some generalizations of this result to some larger Serre subcategories of the category of all R-modules. More precisley, we talk about the following two new generalizations of Kawasaki's conjecture for the classes of minimax and weakly Laskerian R-modules.
(i) Under the assumption $\dim(R/I) = 1$, the category $\mathfrak{M}(R,I)_{com}$ of I-cominimax modules forms an Abelian subcategory of the category of all R-modules. (See [1])
(ii) Under the assumption $\dim(R/I) = 1$, the category $\mathfrak{M}(R,I)_{wcof}$ of I-weakly cofinite modules forms an Abelian subcategory of the category of all R-modules. (See [2])
Finally, we provide some new non-solved problems concerning this area of research.

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Componentwise Polymatroidal Ideals

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All monomial ideals with linear quotients have componentwise linear quotients. It would be of interest to know whether the converse is true. In this talk we introduce componentwise polymatroidal ideals, namely those monomial ideals with the property that each of its components is generated by a polymatroidal ideal. Since componentwise polymatroidal ideals have componentwise linear quotients, it is natural to ask whether componentwise polymatroidal ideals have linear quotients. We expect that this is the case and prove it for ideals which are componentwise of Veronese type.

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Direct Limit of Cohen-Macaulay Rings

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In this paper we study direct system of Cohen-Macaulay rings. We show that direct summands of a direct limit of Noetherian regular domains, that containing a field, are Cohen-Macaulay in the context of non-Noetherian rings. We present several applications of this result. For example, we extend a famous result of Hochster on Cohen-Macaulayness of affine normal semigroup rings. As another application, we study the Cohen-Macaulayness of direct summand of rings of global dimension two.

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On the Faltings' Local-Global Principle

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In this talk, we generalize the Local-global Principle for the annihilation of local cohomology modules. More precisely, let R be a commutative Noetherian ring, M be a finitely generated R-module and $\mathfrak{a}$, $\mathfrak{b}$ be ideals of R with $\mathfrak{b} \subseteq \mathfrak{a}$. It is shown that whenever R is a homomorphic image of a Noetherian Gorenstein ring and $n, s \in \mathbb{N}_0$, then the following statements are equivalent:

1. There exists $t \in \mathbb{N}_0$ such that $\mathfrak{b}^t H^i_{\mathfrak{a}}(M)$ is *'in dimension $< s$'* for all $i < n$;
2. For each $\mathfrak{p} \in \operatorname{Spec} R$, there exists $t_{\mathfrak{p}} \in \mathbb{N}_0$ such that $(\mathfrak{b}R_{\mathfrak{p}})^{t_{\mathfrak{p}}} H^i_{\mathfrak{a}R_{\mathfrak{p}}}(M_{\mathfrak{p}})$ is *'in dimension $< s$'* for all $i < n$.

Note that an R-module T is said to be *'in dimension $< s$'* if there exists a finitely generated submodule K of T such that $\dim \operatorname{Supp} T/K < s$. The special case of equivalence of (1) and (2) in which $s = 0$ is Faltings' Local-global Principle for the annihilation of local cohomology modules.

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A View to Cohomological Dimension of an Ideal

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One of the interesting and long-standing problems in both commutative algebra and algebraic geometry is find out conditions for the equality between height, cohomological dimention, arithmetical rank and analytic spread of an ideal $\mathfrak{a}$ in a local ring $(R, \mathfrak{m})$. In the first part of the talk some conditions for the equality of these invariants will be given. In the light of these results we can take a look at a conjecture on arithmetical rank $\mathrm{ara}(\mathfrak{a})$. Then, with some criteria and examples, we consider the cohomological dimension of an ideal with a view to the set theoretically (cohomologically) complete intersection ideals.

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Relative Derived Categories (I)

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Let $\mathcal{A}$ be an abelian category and $\mathcal{C}$ be a contravariantly finite subcategory of $\mathcal{A}$. One can define $\mathcal{C}$-relative derived category of $\mathcal{A}$, denoted by $\mathbb{D}_{\mathcal{C}}(\mathcal{A})$. This category was first studied by Neeman as a derived category of an exact category [N]. The interesting case for us is when $\mathcal{A}$ has enough projective objects and $\mathcal{C} = \mathcal{GP}$-$\mathcal{A}$ is the class of Gorenstein projective objects. In this case, $\mathbb{D}_{\mathcal{C}}(\mathcal{A})$ is known as a Gorenstein derived category of $\mathcal{A}$ which introduced and verified by Gao and Zhang [GZ].

There are various questions arises naturally in connection to these triangulated categories. Here, we are going to discuss some of these questions.

Let A and B be two rings. A and B are called derived equivalent if there is a triangulated equivalence $\mathbb{D}^b(\text{mod-}A) \cong \mathbb{D}^b(\text{mod-}B)$. A natural question arising from Morita theory is when the bounded derived categories of two rings are equivalent. Rickard in [Ric] obtained necessary and sufficient conditions for two rings to be derived equivalent. Analogously, relative derived equivalence can be defined. In this talk, we present a relative version of Rickard's theorem, specially for Gorenstein derived categories, explain some invariants under Gorenstein derived equivalences and the relationships between relative and absolute derived categories. Finally, we state our relative results related to recollements of derived categories that can be considered as a generalization of (relative) derived equivalence.

This talk is based on two joint works with J. Asadollahi, P. Bahiraei and R. Vahed.

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Gluing of Monomial Curves

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We discuss arithmetic properties of tangent cones associated to large families of monomial curves obtained by gluing. In particular, we characterize their Cohen-Macaulay and Gorenstein properties and prove that they have non-decreasing Hilbert functions. The results come from a careful analysis of some special Apry sets of the numerical semigroups obtained by gluing under a condition that we call specific gluing.

Theses results are based on a joint work with S. Zarzuela Armengou, Departament d' Álgebra i Geometria, Universitat de Barcelona, Gran Via 585, 08007 Barcelona, Spain.

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Quintasymptotic Prime Ideals and Rees Valuation Rings

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Iran

Let I be an ideal of a Noetherian ring R. Let $\mathscr{R} = \mathscr{R}(R, I)$ be the Rees ring of R with respect to I and $\mathfrak{q}$ be a prime ideal of $\mathscr{R}$ such that $\mathfrak{q} \cap R = \mathfrak{p}$. We show that $\mathfrak{q}$ is a quintasymptotic prime ideal of $t^{-1}\mathscr{R}$ if and only if there exists a minimal prime ideal z in R such that $z \subseteq \mathfrak{p}$ and $\mathfrak{p}/z$ is the center of a Rees valuation ring of ideal $I(R/z)$. We also reprove a result concerning when $\overline{I^n}R_S \cap R = \overline{I^n}$ for all $n > 0$.

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On the Stable Set of Associated Prime Ideals of Monomial Ideals and Square-free Monomial Ideals

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Let R be a commutative Noetherian ring and I be an ideal of R. A well-known result of Brodmann [2] showed that the sequence $\{\mathrm{Ass}_R(R/I^k)\}_{k\geq 1}$ of associated prime ideals is stationary for large k. That is, there exists a positive integer k_0 such that $\mathrm{Ass}_R(R/I^k) = \mathrm{Ass}_R(R/I^{k_0})$ for all $k \geq k_0$. A minimal such k_0 is called the *index of stability* of I and $\mathrm{Ass}_R(R/I^{k_0})$ is called the *stable set of associated prime ideals* of I, which is denoted by $\mathrm{Ass}^\infty(I)$. A natural question arises in the context of Brodmann's Theorem:

$(*)$ Is it true that $\mathrm{Ass}_R(R/I) \subseteq \mathrm{Ass}_R(R/I^2) \subseteq \cdots \subseteq \mathrm{Ass}_R(R/I^k) \subseteq \cdots$?
We say that an ideal I of R satisfies *persistence property* if it holds true in $(*)$.

This question does not have a positive answer in general, even for monomial ideals. In recent researches, by using combinatorial tools, many papers have been published in order to describing the stable set of associated prime ideals for a monomial ideal and a square-free monomial ideal (see [1], [3], [5], [7]). Persistence property has been proved for some classes of monomial ideals (see [4], [6], [9]).

In this talk we prove that if $\mathfrak{A} = \{\mathfrak{p}_1, \ldots, \mathfrak{p}_m\}$ and $\mathfrak{B} = \{\mathfrak{p}'_1, \ldots, \mathfrak{p}'_t\}$ are two arbitrary sets of monomial prime ideals of R, then there exist monomial ideals I and J of R such that $I \subseteq J$, $\mathrm{Ass}^\infty(I) = \mathfrak{A} \cup \mathfrak{B}$, $\mathrm{Ass}_R(R/J) = \mathfrak{B}$ and $\mathrm{Ass}_R(J/I) = \mathfrak{A}\backslash\mathfrak{B}$. We show that when $\mathfrak{p}_1, \ldots, \mathfrak{p}_m$ are non-zero monomial prime ideals of R generated by disjoint non-empty subsets of $\{x_1, \ldots, x_n\}$, then there exists a square-free monomial ideal I such that $\mathrm{Ass}_R(R/I^k) = \{\mathfrak{p}_1, \ldots, \mathfrak{p}_m\}$ for all $k \geq 1$, and so $\mathrm{Ass}^\infty(I) = \{\mathfrak{p}_1, \ldots, \mathfrak{p}_m\}$. Also, we introduce another class of monomial ideals with the persistence property.

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The Existence of Relative Pure Injective Envelopes

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In this talk, we study the notion of $\mathcal{S}$-purity introduced by Warfield (Pacific J. Math., **28**(3), 1969, 699-719). For any class $\mathcal{S}$ of R-modules, the class of $\mathcal{S}$-pure flat R-modules is covering. Let $\mathcal{S}$ be a class of finitely presented R-modules such that $R \in \mathcal{S}$ and $\mathcal{S}$ has a subclass $\mathcal{S}^*$, which is a set with the property that for any $U \in \mathcal{S}$ there is a $U^* \in \mathcal{S}^*$ with $U^* \cong U$. Denote the class of all $\mathcal{S}$-pure projective R-modules and the class of all $\mathcal{S}$-pure injective R-modules by $\mathcal{SP}$ and $\mathcal{SI}$, respectively. Then, the class of $\mathcal{S}$-pure injective R-modules is enveloping. In view of the right drived functor Ext in classical homological algebra, we study the right drived functors $\mathrm{Ext}_{\mathcal{SP}}$ and $\mathrm{Ext}_{\mathcal{SI}}$ based on relative homological algebra. In this case, the functor $\mathrm{Hom}_R(-,\sim)$ is right balanced by $\mathcal{SP} \times \mathcal{SI}$. Consequently, $\mathrm{Ext}^n_{\mathcal{SP}}(M,N) \cong \mathrm{Ext}^n_{\mathcal{SI}}(M,N)$ for all R-modules M and N and all $n \geqslant 0$. Accordingly, the global $\mathcal{S}$-pure projective dimension of R is equal to its global $\mathcal{S}$-pure injective dimension.

This is a joint work with Kamran Divaani-Aazar.

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On the Existence of Certain Modules of Finite Gorenstein Homological Dimensions

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Let $(R, \mathfrak{m})$ be a commutative Noetherian local ring. It is known that R is Cohen-Macaulay if there exists either a nonzero finitely generated R-module of finite injective dimension or a nonzero Cohen-Macaulay R-module of finite projective dimension. In this lecture, we investigate the Gorenstein analogues of these facts.

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On Vertex Decomposable Simplicial Complexes and their Alexander Duals

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We study the Alexander dual of a vertex decomposable simplicial complex. The concept of a vertex splittable ideal is defined and it is shown that a simplicial complex Δ is vertex decomposable if and only if $I_{\Delta^\vee}$ is a vertex splittable ideal. Moreover, the properties of vertex splittable ideals are studied. As the main result, it is proved that any vertex splittable ideal has a Betti splitting and the graded Betti numbers of such ideals are explained with a recursive formula. As a corollary, recursive formulas for the regularity and projective dimension of R/I_Δ, when Δ is a vertex decomposable simplicial complex, are given. Moreover, for a vertex decomposable graph G, a recursive formula for the graded Betti numbers of its vertex cover ideal is presented. In special cases, this formula is explained, when G is chordal or a sequentially Cohen-Macaulay bipartite graph. Finally, it is shown that an edge ideal of a graph is vertex splittable if and only if it has linear resolution.

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Jacobian Ideal of Affine Hypersurfaces with Isolated Singularity

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Let $R = k[x_1, \ldots, x_n]$ be the polynomial ring over an algebraically closed filed k of characteristic zero. Let $f \in R$ be a polynomial which defines a reduced hypersurface $X = \mathrm{Spec}\,(R/(f))$. Let $J_f = (f, \partial f/\partial x_1, \ldots \partial f/\partial x_n)$, the so called jacobian ideal of f. The Jacobian ideal J_f defines the singular locus of the hypersurface X. We say that the ideal J_f is of linear type if the symmetric algebra (naive blowup) of J_f is isomorphic with the Rees Algebra $\mathcal{R}_R(J_f)$.

When f defines a smooth hypersurfac then J_f is of linear type. Therefore, smooth hypersurfaces are not interesting for study linear type property. So we assume that X has isolated singularities. In the case of projective hypersurfaces, the ideal J_f is almost complete intersection. Nasrollah Nejad and Simis proved that the Jacobian ideal of f is of linear type if and only if the coordinates of vector filed of $\mathbb{P}_k^{n-1}$ vanishing on f generate an irrelevant maximal ideal and the latter is equivalent to say that locally at each singular prime the Jacobian ideal is a complete intersection [3]. The question motivated this talk is: when the jacobian ideal of an affine hypersurface defined by f is of linear type? An answer to this question is very important in Intersection theory and algebraic geometry. Indeed, If the jacobian ideal of a hypersurface defined by f is of linear type, then the Aluffi algebra $\mathcal{A}_{R/(f)}(J_f/(f))$ is isomorphic with the symmetric algebra of $J_f/(f)$ [3]. By results of Aluffi in [1], we can describe the characteristic cycle of the hypersurface in term of the cycles of the symmetric algebra, in a nutshell, the characteristic cycle of hypersurface $X = \mathrm{Spec}\,(R/(f))$ is

$$(-1)^{\dim X}[\mathrm{Proj}(\mathcal{A}_{R/(f)}(J_f/(f)))] = (-1)^{\dim X}[\mathrm{Proj}(\mathcal{S}_{R/(f)}(J_f/(f)))].$$

In this talk, we prove that the jacobian ideal J_f of an affine hypersurface with isolated singularity is of linear type if and only if f is locally quasi homogenous polynomial, the later is equivalent to say that J_f is locally complete intersection. Finally, we find some classes of locally quasi-homogenous polynomials in $k[x, y]$.

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Cohen-Macaulay of Bigraded Modules

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Let M be a finitely generated bigraded module over the standard bigraded polynomial ring $S = K[x_1, \ldots, x_m, y_1, \ldots, y_n]$ where K is a field. In this talk, we will discuss several Cohen–Macaulayness of M with respect to $Q = (y_1, \ldots, y_n)$, namely, sequentially Cohen–Macaulay, generalized Cohen–Macaulay and sequentially generalized Cohen–Macaulay with respect to Q.

This is a joint work with L. Omarmeli and H. Normohamadi.

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Binomial Ideals Associated to Graphs

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Let G be a finite simple graph with vertex set $[n]$ and edge set $E(G)$. Also, let $S = K[x_1, \dots, x_n, y_1, \dots, y_n]$ be the polynomial ring over a field K. Then the **binomial edge ideal** of G in S, denoted by J_G, is generated by binomials of the form $f_{ij} = x_i y_j - x_j y_i$, where $i < j$ and $\{i, j\} \in E(G)$. One could also see this ideal as the ideal generated by a collection of 2-minors of a $(2 \times n)$-matrix whose entries are all indeterminates. Many of the algebraic properties of such ideals have been already studied. Recently, Ene, Herzog, Hibi and Qureshi introduced the binomial edge ideal of a pair of graphs, as a generalization of the binomial edge ideal of a graph. Let G_1 be a graph on the vertex set $[m]$ and G_2 a graph on the vertex set $[n]$, and let $X = (x_{ij})$ be an $(m \times n)$-matrix of indeterminates. Let $K[X]$ be the polynomial ring in the variables x_{ij}, where $i = 1, \dots, m$ and $j = 1, \dots, n$. Let $e = \{i, j\}$ for some $1 \leq i < j \leq m$ and $f = \{t, l\}$ for some $1 \leq t < l \leq n$. To the pair (e, f), the following 2-minor of X is assigned:

$$p_{e,f} = [i, j|t, l] = x_{it}x_{jl} - x_{il}x_{jt}.$$

Then, the ideal

$$J_{G_1,G_2} = (p_{e,f} : \ e \in E(G_1), f \in E(G_2))$$

is called the **binomial edge ideal of the pair** (G_1, G_2).

We study some of the algebraic properties and invariants of these ideals. We characterize all pairs of graphs (G_1, G_2), for which the binomial edge ideal J_{G_1,G_2} has a linear resolution. We also determine when J_{G_1,G_2} has linear relations. We also compute some of the graded Betti numbers and the Castelnuovo-Mumford regularity of the binomial edge ideal of a pair of graphs with respect to some graphical terms.

This is based on a joint work with Dariush Kiani.

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Relative Derived Categories (II)

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The Auslander-Reiten theory in triangulated categories was first initiated by Happel [Ha1]. He defined Auslander-Reiten triangles and investigated their existence. A known result of Happel [Ha2] states that for a finite dimensional algebra Λ, $\mathbb{D}^{\mathrm{b}}(\text{mod-}\Lambda)$ has Auslander-Reiten triangles if and only if Λ has finite global dimension.

In this talk, we present a Gorenstein version of Happel's result [Ha2, Corollary 1.5], for Gorenstein derived categories. More precisely, we prove that if Λ is a virtually Gorenstein algebra, then $\mathbb{D}^{\mathrm{b}}_{\mathcal{G}p}(\text{mod-}\Lambda)$ has Auslander-Reiten triangles if and only if Λ is Gorenstein. This also provides a generalization of the main result of [G] that states $\mathbb{D}^{\mathrm{b}}_{\mathcal{GP}}(\text{mod-}\Lambda)$ has Auslander-Reiten triangles, provided Λ is a finite dimensional Gorenstein algebra of finite CM-type.

The notion of reflection functors were introduced by Bernstein, Gel'fand, and Ponomarev [BGP] to relate representations of two quivers. Here, we prove that if Λ is a Gorenstein algebra, then $\mathbb{D}^{\mathrm{b}}_{\mathcal{G}p}(\mathcal{Q})$ is equivalent to $\mathbb{D}^{\mathrm{b}}_{\mathcal{G}p}(\sigma_i\mathcal{Q})$ via reflection functors, as triangulated categories. This is a relative version of a result of the absolute one due to Happel [Ha1, I. 5.7].

This talk is based on a joint work with J. Asadollahi, P. Bahiraei and R. Hafezi.

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Two Results on the Regularity of Monomial Ideals

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This talk contains two results on the regularity of a (square-free) monomial ideal. For a given monomial ideal $I \subset K[x_1, \ldots, x_n]$, let $t_i(I)$ denote the maximum degree of a minimal generator in the i-th syzygy of I. We will show that, $t_{i+1}(I) \leq t_i(I) + t_0(I)$. This gives an upper bound for the regularity of a monomial ideal. For the next result, we assume that I is a square-free monomial ideal and $I_{[t]}$ is the ideal generated by all square-free monomials of degree t in I. We will find a formula for the regularity of I, in terms of the regularity of $I_{[t]}$. As an application of this formula, we introduce a procedure for generating ideals with linear resolution.