What Can You Gain from Satisfaction Predicates?

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This talk reports on joint work with Albert Visser (Utrecht). The setting is as follows: we start with a foundational base theory B formulated in a language $\mathcal{L}_B$ (such as $B =$ Peano arithmetic, or $B =$ Zermelo-Fraenkel theory), and then we extend B to a new theory $B^+ := B \cup \Sigma$, where Σ is a set of sentences formulated in the language $\mathcal{L}_B \cup \{S\}$ that captures certain natural features of a Tarskian satisfaction predicate S over an ω-model of B (i.e., a model of B with no nonstandard numbers).

I will give an overview of our current knowledge of the relationship between B and B^+ in connection with the following questions (for various choices of B and Σ).

- Is B^+ *semantically conservative* over B? In other words, does every model of B expand to a model of B^+?

- Is B^+ *syntactically conservative* over B? In other words, if $B^+ \vdash \varphi$, where φ is an $\mathcal{L}_B$-sentence, then $B \vdash \varphi$?

- Is B^+ *interpretable* in B?

- What type of *speed-up* (if any) does B^+ have over B?

The above metamathematical questions, it should added, play a prominent role in the philosophical debate concerning the deflationist conception of truth.