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Moore's Conjecture for (strongly) Gorenstein Flat Modules

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A conjecture of Moore claims that if Γ is a group and Γ' a finite index subgroup of Γ such that $(\Gamma - \Gamma')$ has no elements of prime order, then a $\mathbb{Z}\Gamma$-module which is projective over $\mathbb{Z}\Gamma'$ is projective over $\mathbb{Z}\Gamma$; see [1]. This conjecture is a far reaching generalization of a well-known result of Serre, which asserts that if Γ is a torsion-free group and Γ' a subgroup of finite index, then they have the same cohomological dimension. A result of Aljadeff and Meir [1] proves the validity of Moore's conjecture for all groups inside Krophollers hierarchy $\mathbf{LH}\mathfrak{F}$. In addition, an analogue of this conjecture for injective modules is investigated in [2]. It turned out that the validity of conjecture for injective modules implies the validity of it on projective modules. On the other hand, inspired by the well-known fact that Gorenstein homological modules (Gorenstein projetive, Gorenstein injective and Gorenstein flat modules) share many nice properties of the classical homological modules (projective, injective and flat modules), the authors in [2] also have examined Moore's conjecture for Gorenstein projective and Gorenstein injective modules. Surprisingly, it is proved that in this case Moore's conjecture holds true for all groups, regardless of the hypothesis that $(\Gamma - \Gamma')$ has no elements of prime order. Precisely, it is proved that a given $\mathbb{Z}\Gamma$-module M is Gorenstein projective (Gorenstein injective, resp.) if and only if it is Gorenstein projective (Gorenstein injective, resp.) over $\mathbb{Z}\Gamma'$, whenever Γ' is a subgroup of Γ of finite index. Here, we study Moore's conjecture for (strongly) Gorenstein flat modules. The notion of strongly Gorenstein flat modules has been introduced and studied by Ding et al. [4]. A $\mathbb{Z}\Gamma$-module M is said to be strongly Gorenstein flat if there exists an exact sequence of projective $\mathbb{Z}\Gamma$-modules

$$\mathbf{P}_\bullet : \cdots \longrightarrow P_{n+1} \xrightarrow{\delta_{n+1}} P_n \xrightarrow{\delta_n} P_{n-1} \longrightarrow \cdots ,$$

with $M = \operatorname{Im}\delta_0$ and such that the functor $\operatorname{Hom}_{\mathbb{Z}\Gamma}(-, F)$, where F is a flat $\mathbb{Z}\Gamma$-module, leaves the sequence exact. It is worth noting that this definition is different from the concept of strongly Gorenstein flat modules studied in [3]. We show that the counterpart of Moore's conjecture for Gorenstein projective and injective modules, remains true for Gorenstein flat and strongly Gorenstein flat modules. Moreover, it will turn out that the invariant strongly Gorenstein flat dimension of the trivial $\mathbb{Z}\Gamma$-module $\mathbb{Z}$ has enough potential in reflecting the properties of the underlying group.

We should point out that although it follows from the definition that any strongly Gorenstein flat module is Gorenstein projective, it is unknown that whether these notions

are the same. In the setting of group rings, we provide some criteria in which these two notions coincide.

Keywords: Moore's conjecture, Gorenstein projective modules, strongly Gorenstein flat modules, group ring.

References

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