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An Investigation on Hankel Matrix

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The motivation of this work starts from the following question:

Question 0.0.1 *Let $\mathcal{M}$ denote a square matrix over $k[\mathbf{x}] = k[x_0, \cdots, x_n]$ whose entries are forms of equal degrees and $f = \det(\mathcal{M})$. For which matrices f is a homaloidal determinant?*

It looks like the question is wide open even if the entries are linear.There is a class of examples in which the answer to the question is affirmative. We are going to show that the answer of this question is negative in the case of Hankel matrices. The theme of this lecture starts from an investigation of the polar map defined by determinant of Hankel Matrix, say f. The focus is on the homological properties of the gradient ideal ∇f as a method of studying its homaloidal behavior. In characteristic zero we show that ∇f is defining a dominant map. By algebraic tools we show some relations between determinantal ideals of Hankel matrix and ∇f. We have used arguments ranging from multiplicities to initial ideals to Plücker relations(straightening laws) of Hankel maximal minors via the Gruson-Peskine change of matrix trick. The linear syzygies and unmixed part of the representative ideal of Hankel determinant's polar map are well-studied. Geometrically gradient ideal of f is $I_{n-1}(H) = P$, actually it is a minimal reduction of prime ideal P. As a conjecture of authors J is of linear type, more explicitly $\nabla f P^i : P^{i+1} = I_{n-2-i}(H)$ for $0 \leq i \leq n-2$. Consequently f is not a homaloidal polynomial.