Computing Local Cohomology by Using Spectral Sequences

Lecture I: The Homological Case
Lecture II: The Cohomological Case

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Spectral sequences are often used to compute local cohomology functors. In these two talks we are going to consider how to use them in order to compute local cohomology from the primary decomposition of an ideal \( I \) in a commutative Noetherian ring \( R \). In the homological case, we shall deal with the computation of several generalized local cohomology functors supported on \( I \). In the cohomological case we shall mainly be concerned with the computation of the local cohomology of \( R/I \). The construction of these spectral sequences is done by means of the computation of the left and right derived functors of the inverse limit in terms of the homology (or cohomology) of a particular explicit homological (or cohomological) complex. In each case, we shall also give sufficient conditions in order to assure the degeneration of the corresponding spectral sequence and study the extension problems arising from the associated filtrations. As a guiding cases we will have in mind the results obtained by Alvarez-Garcia-Zarzuela in 2003 and G. Lyubeznik in 2007 in the homological case, and the well known Hochsters formula for the local cohomology of Stanley-Reisner rings in the cohomological case. These talks is based on joint work with Josep Alvarez Montaner and Alberto F. Boix.