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Toroidalization of Locally Toroidal Morphisms of 3-Folds

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The idea of toroidalization is fundamental to study the structure of birational morphisms in algebraic geometry. Given a dominant morphism of algebraic varieties $\varphi : X \to Y$, over an algebraically closed field $\mathfrak{k}$ of characteristic zero, the toroidalization problem is that of obtaining sequences of blow-ups with nonsingular centers $\lambda : \widetilde{X} \to X$ and $\pi : \widetilde{Y} \to Y$ so that we can achieve a commutative diagram

$$\begin{array}{ccc} \widetilde{X} & \xrightarrow{\tilde{\varphi}} & \widetilde{Y} \\ {\scriptstyle\lambda}\downarrow & & \downarrow{\scriptstyle\pi} \\ X & \xrightarrow[\varphi]{} & Y \end{array}$$

such that $\tilde{\varphi}$ is a toroidal morphism (locally given by monomials in appropriate *é*tale local parameters on $\widetilde{X}$, with respect to fixed SNC divisors $D_{\widetilde{X}}$ and $D_{\widetilde{Y}}$ on $\widetilde{X}$ and $\widetilde{Y}$ respectively).

The existence of toroidalization has been proved completely when Y is a curve or when X and Y are of dimension $\leqslant 3$ by Professor S. D. Cutkosky. Also, it has been proved for strongly prepared morphisms from a nonsingular n-fold X to a nonsingular surface S by Professor Cutkosky and Kashcheyeva, and for locally toroidal morphisms from a nonsingular n-fold to a nonsingular surface S by Hanamanthu. In this talk we will present an introduction to the problem of toroidalization and we will discuss our recent proof of toroidalization of locally toroidal morphisms of 3-folds.