Postulation of Disjoint Unions of Lines and a Triple Point

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A scheme $X \subset \mathbb{P}^n$ is said to have *good postulation* if for all integers $d > 0$ the restriction map $\rho(d) : H^0(\mathbb{P}^n, \mathcal{O}_{\mathbb{P}^n}(d)) \to H^0(X, \mathcal{O}_X(d))$ has maximal rank, i.e. if X imposes the expected number of conditions on degree d hypersurfaces in $\mathbb{P}^n$. R. Hartshorne and A. Hirschowitz proved that a generic collection of lines in $\mathbb{P}^n$, $n \geq 3$, has good postulation [5]. E. Carlini, M. V. Catalisano and A. V. Geramita considered the postulation problem for a general union $X \subset \mathbb{P}^n$ of skew lines and one m-point [4]. They proved that when $n \geq 4$ the scheme X has good postulation, but when $n = 3$ they conjectured that there are some exceptional cases, in particular they proved their conjecture for $m = 2$ [4]. In this paper we prove their conjecture in the case $m = 3$, i.e. we prove the following theorem.

Theorem 0.1. *Let $X \subset \mathbb{P}^3$ be a general union of t disjoint lines and one triple point. X has good postulation if and only if either $t = 1$ or $t \geq 4$.*

This talk is based on a joint work with Edoardo Ballico.

References

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