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Triangular Decompositions of Polynomial Systems: Reverse Engineering from Wu to Ritt through

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Triangular decompositions are one of the major tools for solving polynomial systems. For systems of algebraic equations, they provide a convenient way to describe complex solutions and a step toward isolation of real roots or decomposition into irreducible components. Combined with other techniques, they are used for these purposes by several computer algebra systems. For systems of partial differential equations, they provide the main practicable way for determining a symbolic description of the solution set. Moreover, thanks to Rosenfeld's Lemma, techniques from the algebraic case apply to the differential one.

Research in this area is following the natural cycle: *theory, algorithms, implementation*, which will be a motto of this tutorial. However, while the subject originated in differential algebra with the work of Riit before being adpated to polynomial algebra by Wu and his followers, we shall take the opposite way, in five lectures.

The first two provide an overview of the theory of regular chains, which was introduced by Kalkbrener. We shall see how regular chains can be used to represent and manipulate algebraic varieties, constructible sets and semi-algebraic sets. The RegularChains library (available at www.regularchains.org) will illustrate this pre- sentation.

In the third lecture, we will discuss adaptations of the theory of regular chains to the differential case. In the fourth lecture, we will apply the theory of regular chains to the study of dynamical systems, Examples will range from biological networks to Hilbert's sixteenth problem. Complexity results and implementation issues will be the topics of the fifth lecture.