On Cellular Resolution of Monomial Ideals
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Let \( I \subset R = k[y_1,\ldots,y_n] \) be a monomial ideal in the polynomial ring over a field \( k \) and let \( G(I) \) be the unique minimal monomial generating set of \( I \). Let \( X \) be a regular cell complex with \( G(I) \) as its vertices. Let \( \epsilon_X \) be an incidence function on \( X \). Any face of \( X \) will be labeled by \( m_F \), the least common multiple of the monomials in \( G(I) \) which correspond to the vertices of \( F \). If \( m_F = y_1^{a_1}\ldots y_n^{a_n} \), then the degree \( a_F \) is defined to be the exponent vector \( e(m_F) = (a_1,\ldots,a_n) \). Let \( RF \) be the free \( R \)-module with one generator in degree \( a_F \). The cellular complex \( F_X \) is the \( \mathbb{Z}^n \)-graded \( R \)-module \( \bigoplus_{\emptyset \neq F \in X} RF \) with differentials

\[
\partial(F) = \sum_{\emptyset \neq F' \in X} \epsilon(F,F') \frac{m_F}{m_{F'}} F'
\]

If the complex \( F_X \) is exact, then \( F_X \) is called a cellular resolution of \( I \). Alternatively, we say that \( I \) has a cellular resolution supported on the labeled cell complex \( X \). If \( X \) is a polytope or a simplicial complex, then \( F_X \) is called polytopal, and simplicial, respectively.

The idea to describe a resolution of a monomial ideal by means of combinatorial chain complexes was initiated by Bayer, Peeva and Sturmfels [1], and was extended by Bayer and Sturmfels [2], and further extension was made by Jöllmebeck and Welker [6]. Further contributions were given by Sinetakoupols [8], Mermin [7], Dochtermann and Engström [3],[4] and Goodarzi [5].

We consider monomial ideals that are facet ideals of a linear matroid of a set of vectors on which the only linear dependence is proportionality, i.e., some vectors could be scaler multiples of others. We prove that any such matroidal ideal has a linear resolution supported on a polytopal cell complex.

References


