## Investing the Efficiency of Monte Carlo in Option Pricing under Jump-Diffusion Models

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In 2008, Giles [M. B. Giles. Multilevel Monte Carlo path simulation, Operations Research, 56(3):607617, 2008.] showed that we can reduce the computational cost of estimating the expected value of the payoff depending upon the solution of a stochastic differential equation (SDE) by applying the Multilevel Monte Carlo Method (MLMC). In this paper we use MLMC for pricing an option under jump-diffusion dynamics. Numerical results reveal a significant reduction in computational cost.

## Measuring Systemic Risk by CoVaR approach in Tehran Stock Exchange

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Systemic risk explores the probability of the fall of the whole system. Investors usually care about losing the value of a share or commodity while in systemic risk the emphasis is on the whole market. This falling often takes place when a key firm faces a big drop in a financial system. The fear of bankruptcy causes a negative effect on the other companies like domino and put them on a declining way. These chain reactions may impose stress to the market and push it to a real crisis. The motivation of this article is to review of the history of systemic risk modeling and introduce a number of common systemic risk measures. Generally, systemic risk is classified to two categories: 1. the measures that evaluate the risk of the whole system when a firm is in distress and 2. the measures which measure the risk of a firm when the whole system is in distress. This article uses the quantile regression to measure systemic risk for 20 companies in Tehran stock exchange by Conditional Value at Risk CoVaR approach.

# Superonvergnce of the Finite Element Solutions to Price Discrete Double Barrier Options under a CEV Model with Jump Diffusion

### Davood Ahmadian

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We develop a numerical method to price discrete barrier options on an underlying described by the constant elasticity of variance model with jump-diffusion (CEVJD). In particular, the partial integro differential equation associated to this model is discretized in time using an operator splitting scheme whose accuracy is enhanced by repeated Richardson extrapolation. Such an approach allows us to approximate the differential terms and the jump integral by means of two different numerical techniques. Precisely, the spatial derivatives, which exist only in the weak sense, are discretized using a finite element method (FEM) based on piecewise quadratic polynomials, whereas the jump integral is directly collocated at the mesh points, so that it can be easily evaluated by Simpson numerical quadrature. As shown by extensive numerical simulation, the proposed approach is very efficient from the computational standpoint, and performs significantly better than the finite difference scheme developed in related papers.

## A Continuous Time Speculative Storage Model for Pricing Agricultural Insurances

#### Hirbod Assa

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The rational expectations model determines the optimal inventory decisions. The basic version of the model developed by Deaton and Laroque (1992, 1995, 1996) incorporates competitive storage into the consumer demand and supply dynamics and establishes the concept of stationary rational expectations equilibrium. Their model was studied and extended from different aspects, in order to capture some stylized facts observed in the actual data. However, in all these studies the model is in a discrete time framework, which does not allow using proper techniques in the continuous time settings. In this paper for the first time we will introduce a continuous time Deaton and Laroque model and use the existing tools from financial engineering which have been developed in the derivative pricing literature to price the insurance products on agricultural goods.

## On Optimal Reinsurance Policy with Distortion Risk Measures and Premiums

## Hirbod Assa

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In this paper, we consider the problem of optimal reinsurance design, when the risk is measured by a distortion risk measure and the premium is given by a distortion risk premium. First, we show how the optimal reinsurance design for the ceding company, the reinsurance company and the social planner can be formulated in the same way. Second, by introducing the "marginal indemnification functions", we characterize the optimal reinsurance contracts. We show that, for an optimal policy, the associated marginal indemnification function only takes the values zero and one. We will see how the roles of the market preferences and premiums and that of the total risk are separated.

## **Jointly Optimal Reinsurance**

## Ahmad Bigdeli

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Several approaches to optimal reinsurance have been attempted in the actuarial literature. A common feature of most of the quoted works is that optimality is considered with respect to the interest of solely the direct insurer. In the present paper, optimal reinsurance is considered from the point of view of both the interests of the primary insurer and the reinsurer, as two parties jointly liable for the risk they share. The problem of optimal excess of loss reinsurance with a limiting and a retention level is considered. It is demonstrated that this problem can be solved, combining specific risk and performance measures, under some relatively general assumptions for the risk model, under which the premium income is modeled by any nonnegative, nondecreasing function, claim arrivals follow a Poisson process and claim amounts are modeled by any continuous joint distribution. As a "performance measure", "the expected profits of the direct insurer and the reinsurer, given their joint survival" is defined. "The probability of joint survival of the direct insurer and the reinsurer" is employed as a "risk measure". Finally, the optimal excess of loss reinsurance treaty is obtained at first with use of risk measure only, and then with a combination of two introduced criteria.

## Model Risk in Generalizing the Black-Scholes Model

Hassan Dadashi (two sessions)

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The celebrated Black-Scholes model fails to capture the main characteristics of the actual stock prices dynamics, such as the skewness and the kurtosis. There are a few extensions of this model which have been proposed in the literature, among which the jump diffusion models as well as the stochastic and the local volatility models have drawn more attention. The major risk of utilizing these extensions is the goodness of their fit to the actual data, known as the model risk. In this short lecture, by using the Bayesian methodology, we introduce some goodness of fit measures for the Black-Scholes model extensions, and try to reduce the model risk by studying these measures.

# Pricing of Boundary Linked assets by Stochastic Boundary Value Problems Solved with a New Adaptive Multiple Shooting Methods

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With the growing sophistication of financial markets, investors are demanding new, more complex options products, tailored to their needs. In particular, there is an increasing number of financial assets whose values are contractually linked at certain periods of time, such as leases and rental agreements. An illustrative example is the English real estate lease market. The value of lease assets can be formulated by a secondorder boundary value stochastic differential equation SDEBVP:

 $X'(t)=b(t)X(t)dt+W(t), 0 \le t \le T, h(X(0),X(T)) = \beta.$ 

that is, the acceleration of lease assets prices is proportional to their growth rate and affected by a white noise shock. Hence, to value boundary linked assets, we are faced with the problem of solving stochastic differential equations with boundary conditions which, often, is not Markovian. An adaptive multiple shooting method is proposed for solving the stochastic boundary problem. We illustrate the effectiveness of this approach on several standard test problems selected from the literature and compare it with other existing methods. We apply these ideas to study the valuation of boundarylinked assets and their derivatives. Furthermore, we value boundarylinked derivatives using Malliavin calculus and Monte Carlo methods. We apply these ideas to value European call options of boundary linked assets.

## The Optimal Reinsurance and Investment Strategies in an OU Model

# Ramin Eghbalzadeh

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Considering recent progress in insurance industry and its role in economic development, studying the optimal investment and reinsurance strategy is crucial. The object of this thesis is to maximize the expected exponential utility with respect to the terminal wealth. Also with the simulation of diagrams and calculate the sample variance the Brownian motion and Ornstein -Uhlenbeck, we have attempted to compare the two processes together. Assuming that the instantaneous rate of investment return to be given by Ornstein-Uhlenbeck process, we have studied the optimal strategy for the compound Poisson and Brownian motion risk models. Stochastic control theory and Hamilton-Jacobi-Bellman equations are used as the two basic tools. Furthermore an optimal strategy is considered in the case of partial observation.

## **Optimal Stopping and Backward Stochastic Differential Equations with Obstacles**

#### Neda Esmaeeli

Sharif University of Technology, Tehran, Iran

We are concerned with backward stochastic differential equations (BSDEs) with reflecting Obstacles (or barriers) and their connection with the value function of American contingent claims and game contingent claims. The talk will contain two parts: In the first part we will introduce the concept of game contingent claims (GCC in brief) with a short review of American contingent claims in a general framework. From a financial point of view, a game contingent claim could be seen as a generalization of the notion of American contingent claims, where not only has the buyer(or holder) the right to exercise at any time before the maturity, but also the seller(or issuer) has the right to cancel the contract at any time before the maturity time. This is a typical optimal stopping game which is known as a zerosum Dynkin game which was introduced by Dynkin as an extension of the optimal stopping problems. In the second part, we will place ourselves in the framework of backward stochastic differential equations with obstacles. The theory of BSDEs was introduced by Bismut [1973] for the linear case and by Pardoux and Peng [1990] in the general case. It became now very popular, and is an important field of research due to its applications in mathematical finance and stochastic optimization. At beginning, we will present an introduction to the reflected backward stochastic differential equations (RBSDEs in brief) that is the case where the solution of the BSDE is forced to stay above a given stochastic process, called the obstacle and to the Doubly Reflected BSDEs (DRBSDEs in brief) which is the case the solution process of our BSDE has to remain between two prescribed upper and lower boundary processes. Moreover, we will state some important results about the connection of the solutions of RBSDEs and DRBDSEs with the value functions of optimal stopping problems and optimal stopping games respectively. Finally we will mention some questions relevant to this subject.

## **Knightian Uncertainty in Pricing Financial Securities**

## Arash Fahim

Florida State University, Tallahassee, Unites State of America

Distinguished by Frank Knight (1885–1972), some sort of uncertainty cannot be measured, in particular by the means of probability. Here we provide some examples of Knightian uncertainty and explain how they can lead to model uncertainty, with focus on exotic options in discrete-time market. We briefly review two approaches toward incorporating Knightian uncertainty into pricing. First, as suggested by Dupire in 1994, one can calibrate models to market prices of liquid options (such as vanilla options), and then uses the calibrated models to price other illiquid exotic options. Yet, with different models, this calibration procedure may generate different prices for an exotic option. Finding model-independent bounds for prices of exotic options is therefore of practical importance, and it naturally leads to the problem of model-independent super-hedging (or sub-hedging). Second, one can focus on a set of plausible models (not necessarily calibrated to all market data) and seek for the super-hedging (or sub-hedging) bound. The former leads to the notion of model-independent arbitrage, while the later suggests a rather different quasi-surely arbitrage. Fundamental theorem of asset pricing appears in a different shape in each approach. While the second approach can be regarded as a generalization of model-based pricing, model-independent approach has a different nature comparing to the classical model-based pricing.

## **Regime Switching Models in Financial Mathematics: Theory and Numerics**

## Ali Foroush Bastani, Maryam Vahid

Institute for advanced studies in basic sciences, Zanjan, Iran

The Markovian regime switching paradigm has become one of the prevailing models in mathematical finance. It is now widely known that under the regime switching model, the market is incomplete and so many important questions of finance in this framework will be challenging and of considerable importance for market practitioners and academia. Our aim in this talk is to give an overview about the importance and necessary mathematical background in using these models especially when the parameters of a Levy process are considered to have regime shifts. We present some illustrative numerical examples highlighting the potentials of the model to capture a wide variety of observed stylized facts in the market.

## **Monte Carlo Methods in Financial Engineering**

Ali Foroush Bastani, Maryam Vahid (two sessions)

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Monte Carlo methods are used in mathematical finance to value and analyze complex instruments, portfolios and investments by simulating the various sources of uncertainty affecting their value, and then determining their average value over the range of resultant outcomes. This is usually done by help of stochastic asset models. The advantage of Monte Carlo methods over other techniques increases as the dimensions (sources of uncertainty) of the problem increase. In this talk, we start from the basic facts and will describe some general frameworks in which the method could be employed to solve some basic problems of financial engineering like option pricing and evaluation of risk measures.

## The Evaluation of Venture Capital as an Installment Option

Ali Foroush Bastani, Hamed Hamedinia

University of Economic Sciences, Tehran, Iran IASBS, Zanjan, Iran

An installment option is a Bermudan option in which the premium, instead of being paid upfront is paid in series of installments. If all installments are paid, the holder will receive the payoff at maturity time. However, the holder has the right to terminate payments on any payment date, in which case the option lapses with no further payments on either side. Several discrete dates are considered in the Bermudan option in which the holder has the right to exercise the option just at those times (in contrast to American option that holder can exercise it along the life time). In this article, the lower and upper bounds based on "No Arbitrage" condition are obtained for pricing installment option in the discrete and continuous time. These bounds have been used for pricing Venture Capital contract. With Monte Carlo simulation and generation of stock price following GBM, they have been compared with DTS bounds (the bounds obtained by other authors). Results reveal the bounds in the present study are more effective than DTS bounds. Finally, in the continuous time and with using real data, these bounds have been compared with closed form solution (obtained by Wystup based on Black-Scholes assumptions) as well. Moreover, reliability of these assumptions like approximation of volatility and accuracy of GBM are discussed.

## **Applications of Phase Type in Actuarial Science**

Amin Hassanzadeh (three sessions)

Shahid Beheshti University, Tehran, Iran

A phase type ( or PH) random variable, which is defined as the time until absorption in a continuous time Markov chain, is fully characterized by two sets of parameters from that Markov chain: the initial probability vector and transition intensity matrix. In this presentation, in addition to a review of PH models and their properties, the applications of phase type distributions in credibility theory, disability insurance modeling and bivariate losses in insurance are considered. The advantages of PH models over the classical ones will be discussed via some real and simulated data.

#### **Efficient Simulation of Nonlinear Parabolic SPDEs with Correlated Noise**

## Minoo Kamrani

Razi University, Kermanshah, Iran

Stochastic partial differential equations (SPDEs) have many applications in many fields, for example continuum physics, finance. In this paper we consider pathwise numerical approximation of nonlinear parabolic SPDEs driven by additive white noise. We consider the case the Brownian motions are not necessarily independent. A numerical method will be proposed and analyzed. The rate of convergence in uniform topology is estimated for the stochastic Burgers equation. Numerical examples illustrate the estimated convergence rate.

#### Asymptotic Expansion of the American Call Option with Dividends Close to Expiry

Mehdi Dehghan, Ali Foroush Bastani, Seyed Mohammad Mahdi Kazemi

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This paper is concerned with the pricing of American call options close to expiry using an analytic approach. We use the heat equation analogue of the Black-Scholes partial differential equation defined on an unbounded domain and decompose it into inner and outer problems. Using the methodology of Han and Wus approach [H. Han and X. Wu, A fast numerical method for the Black-Scholes equation of American options, SIAM J. Numer. Anal. 41 (6) (2003) 20812095] one could obtain a singular transparent boundary condition (TBC) suitable to solve the inner problem numerically. We pursue an alternate approach by focusing on the outer problem in conjunction with an equivalent nonsingular version of the TBC which is more tractable for analytical purposes. We obtain a closed form expansion along the TBC near the expiry of the option. Using the Poincare asymptotic expansion, we are able to find simple and more computationally efficient approximations to the option price when the risk free interest rate is less than or equal to the dividend yield. Our numerical experiment shows the effectiveness of this approach in comparison with some existing results.

## Strategic Asset Allocation for a Long Run Investment

Chiman Mohammadnejad, Amir T. Payandeh

Shahid Beheshti University, Tehran, Iran

The problem of strategic asset allocation for a long run investment is always the main concern for all institutional investors, such as insurance companies. Time interval between receiving premiums and paying claims, unused and available surplus funds in insurance companies must be utilized properly. This thesis begins modeling of economic cycles' impact on asset returns distribution using copula theory with hidden Markov model. Then, for two static and dynamic strategies, using the transition probabilities and sojourn in each state of the hidden Markov chain a certain investment policy with the highest efficiency will be suggested. By a Monte Carlo simulation study efficiency of two static and dynamic strategies are compared. The simulation study shows that a dynamic strategy is more efficient whenever it uses information about the probability of transmission.

#### Stochastic Euler Approximation for the CIR Model of Interest Rate

<u>Hamidreza M.Almani</u>, Bijan Z.Zangeneh

Sharif University of Technology, Tehran, Iran

In 1985 Cox, Ingersoll and Ross developed a model for interest rate, we now call "CIR Model". They provided the term structure of interest rate as a random variable r(t) which changes according to a particular the stochastic differential equation. The diffusion coefficient of this equation is not in general Lipschitz continuous, so the existence and uniqueness of solution could be an important problem in this case. According to the form of the equation's coefficients, we shall first develop the Euler approximations for the stochastic differential equation with Holder continuous diffusion coefficient and proof the convergence of Euler method in this case. Then with the results we can talk about the existence and uniqueness of the solution for CIR model and even approximate the solution with an appropriate simulation.

## **Credit Risk Management and High Order Risk Measures**

#### Shahab Nankali

Mellat Bank, Tehran, Iran

Default risk is the uncertainty of a firm's ability and willingness to perform its debts and obligations. Prior to default, there is no way to determine firms that will default and those that will not. We can just make probabilistic assessments of the likelihood of default. Default is a rare event. The typical firm has a default probability of around 2% in any year. However, there is considerable variation in default probabilities across firms. Credit Rating is a key word to determine firms' creditworthiness and it can help lenders to make best decision to accept or reject loan applications. As a result, Banks have to evaluate their customers' creditworthiness and measure portfolio credit risk based upon some risk measure such as VaR, CVaR and Expected shortfall, also we propose high order expected shortfall as a measure for credit crisis. This measure can prevent rare event of big insolvency in bank.

## **Bonus-Malus Systems**

Amir Teymour Payandeh (three sessions)

Shahid Beheshti University, Tehran, Iran

Bonus-Malus systems, say BMS, are one of the most important part of insurance industry. This short workshop aims to review actuarial concept of the BMS and answer to question that "How one may design an appropriate BMS?" To answer the above question several comparison methods have been explored.

## **Introduction to Cash Management Models in Banking Systems**

Jamileh Peykar, Hedi Yousefi, Mahdi Mohammadzadeh Monfared, Mehran Farahikia

Ayandeh Bank, Tehran, Iran

Managing liquidity has always been tricky. It's important to have a liquid reserve that will cover expenses and leave a significant margin of error. However, carrying too much cash can represent a lost opportunity for longer term investments. In this paper, after reviewing the Baumol, Miller Orr, and Emery's lambda cash management models, differences among those models are identified, and the best time for applying each of this models to obtain acceptable results are explained. Use of the Baumol Cash Management Model requires that managers be able to forecast cash demand with some degree of accuracy. The Miller Orr model helps companies to meet its cash requirements at the lowest possible cost by placing upper and lower limits on cash balance. The Lambda index helps forecast whether a company will have adequate cash and credit to survive or not, and become insolvent. By this model we will predict the expected cash for some days in the future based upon the cash in some last days.

# **Option Pricing by Using of Functional Perturbation Method**

Somayeh Pourghanbar, Mojtaba Ranjbar

Azarbaijan Shahid Madani University, Tabriz, Iran

In this article we introduce functional perturbation method (FPM) for pricing of barrier option. The FPM is a powerful tool for analytical analysis of linear problems. We first expand the BS equation functionally by Frechet series, yielding a set of partial differential equations (PDEs). For associated PDEs, we use the variational iteration method (VIM). The accuracy and validity of the method is checked by comparison with the exact solution.

## Carma Process Driven by a Regularly Varying Levy Archimedean Copula

## Mohsen Rezapour

Shahid Bahonar University, Kerman, Iran

In this paper, we first introduce Levy Archimedean copula that can be obtain by an Archimedean copula with a dmonotone generator. Then, we show that if the marginal's of the Levy Archimedean copula are regularly varying with positive jumps then it is a regularly varying random process. In last we use this process to construct the continuous time autoregressive moving average (CARMA) process driven by regularly varying Levy Archimedean copula process and investigate its properties.

# **Topics in Fundamentals of Mathematical Finance**

Shiva Zamani (three sessions)

Sharif University of Technology, Tehran, Iran

In this short course we will briefly review the fundamentals of the mathematical finance, including the derivative pricing models, term structure models and the Greek letters.