The 2nd School on Analysis and PDE
May 19-21, 2015, IPM, Tehran, Iran

A Three Lecture Series on the Regularity of Free Boundaries
Part I: What is a Free Boundary

John Andersson
Royal Institute of Technology, Sweden

Talk 1

In this first lecture we will discuss what a free boundary is and introduce some classical free boundary problems. We will show basic existence theory and elementary regularity theory. I will do my best to keep the presentation independent form any advanced results from partial differential equations (PDE) and variational analysis in the hope that anyone with rudimentary knowledge of PDE and calculus of variations should be able to follow.

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A Three Lecture Series on the Regularity of Free Boundaries
Part II: Classical Free Boundary Regularity Theory

John Andersson
Royal Institute of Technology, Sweden

Talk 2

In the second lecture I will focus on the classical regularity theory as developed by Luis Caffarelli. Caffarelli's ideas are very geometrical and elegant. We will try to keep the level so that a someone with a basic masters course in elliptic PDE should be able to follow most of the lecture.

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A Three Lecture Series on the Regularity of Free Boundaries
Part III: Analytical Methods and Linearization

John Andersson
Royal Institute of Technology, Sweden

Talk 3

In the final lecture we will shift the focus from the geometric methods of Caffarelli to more analytic methods and linearization techniques.Several different problems will be discussed and we will focus more on the linearization approach than on technical details.

This talk will be based on research by myself in collaboration with Henrik Shahgholian, Georg Weiss and Erik lindgren.

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Semiclassical Limit of Quantum Liouville Equation

Hassan Emamirad
IPM, Iran

We will show that Liouville and quantum Liouville operators

$$Lw = -\xi.\nabla_x w + \nabla_x V.\nabla_\xi w$$

and

$$L_\hbar w = -\xi \cdot \nabla_x w + P_\hbar(x, \nabla_\xi) w$$

where $P_\hbar$ is a pseudo-differential operator defined in symbolic form by

$$P_\hbar(x, \nabla_\xi) = \frac{\mathrm{i}}{\hbar}\left[V(x + \mathrm{i}\frac{\hbar}{2}\nabla_\xi) - V(x - \mathrm{i}\frac{\hbar}{2}\nabla_\xi)\right],$$

V being the potential of Schrödinger equation, generate two C_0-groups e^{tL} and $\mathrm{e}^{tL_\hbar}$ of isometries in $L^2(\mathbb{R}^n_x \times \mathbb{R}^n_\xi)$ and $\mathrm{e}^{tL_\hbar}$ converges ultraweakly to e^{tL} as $\hbar \to 0$. As a consequence we show that the Gaussian mollifier of the Wigner function, called Husimi function, converges in $L^1(\mathbb{R}^n_x \times \mathbb{R}^n_\xi)$ to the solution of the Liouville equation.

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An Introduction to Mean-Field Games

Diogo Aguira Gomes
Instituto Superior Técnico, Portugal

Talk 1

In this talk we give a short introduction to mean-field games. This will include a brief discussion of (deterministic and stochastic) optimal control and transport and Fokker-Planck equations.

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Existence of Strong Solutions for Mean-Field Games

Diogo Aguira Gomes
Instituto Superior Técnico, Portugal

Talk 2

We will present some recent techniques to prove the existence of smooth solutions through a-priori estimates and continuation methods.

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Existence of Weak Solutions for Mean-Field Games

Diogo Aguira Gomes
Instituto Superior Técnico, Portugal

Talk 3

In the present talk, we discuss monotonicity methods for mean-field games. In particular, we suggest a new definition of weak solution, whose existence can be proven under quite general assumptions. Moreover, we discuss various applications to the numerical approximation of mean-field games.

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The Eigenvalue Problem for the p-Laplacian

Erik Lindgren
Royal Institute of Technology, Sweden

Talk 1

In my first talk I will introduce the eigenvalue problem for the p-Laplace equation. I will discuss basic properties of ground states and also the fascinating limit as $p \to infty$. Some recent progress and open problems will also be presented.

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A Doubly Nonlinear Evolution for Ground States of the p-Laplacian

Erik Lindgren
Royal Institute of Technology, Sweden

Talk 2

In this second talk, I will discuss the evolutionary equation

$$|v_t|^{p-2}v_t = \Delta_p v.$$

A remarkable feature for this equation is that the p-Rayleigh quotientis nonincreasing in time. Properly rescaled, the solutions converge to a first p-eigenfunction as t tends to ∞. An interesting limiting equation also arises when p tends to infinity, related to the infinity eigenvalue problem.

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A Doubly Nonlinear Evolution for Ground States of the p-Laplacian

Erik Lindgren
Royal Institute of Technology, Sweden

Talk 3

In this talk I will introduce the p-Laplace equation, the nonlinear counterpart to the well known Laplace equation. Some basic properties will be explained such as existence of solutions, different notions of solutions and regularity. Some words about the interesting limit when p goes to infinity will also be said.

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Talk 1:
The Asymptotic of the Minimizers of the Mumford-Shah Functional Near the Crack-Tip: Regularity
Talk 2:
The Asymptotic of the Minimizers of the Mumford-Shah Functional Near the Crack-Tip: Numerical Analysisity

Hayk Mikayelyan
Xi'an Jiaotong-Liverpool University, China

In two talks we consider the Mumford-Shah functional in the plain and study the asymptotics of the solution near the crack-tip. This problem arise in certain models of fracture mechanics. It is well-known that the leading term in the asymptotics at the crack-tip is related to the stress intensity factor. We calculate the higher order terms in the asymptotic expansion, and show that those are related to the curvature of the crack close to the tip. We develop new numerical methods to compute the minimizers and verify numerically some open conjectures.

A joint work with John Andersson (KTH Stockholm) and Zhilin Li (North Carolina State University).

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The Free Boundary of Variational Inequalities with Gradient Constraints (I & II)

Mohammad Safdari
IPM, Iran

In these talks we show that the free boundary of some variational inequalities with gradient constraints is as regular as the part of the boundary that parametrizes it. To this end, we study a generalized notion of ridge of a domain in the plane, which is the set of singularity of some distance function to the boundary of the domain.

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Functional and Geometric Inequalities in Sobolev Spaces: From Isocapacitary Estimates to Littlewood-Paley Theory (I & II)

Ali Taheri
University of Sussex, UK

I begin the lectures by giving a brief survey on various geometric and Sobolev-type inequalities and highlighting the role of entropy monotonicity and isocapacitary estimates in establishing such inequalities. In the second lecture I will move on to presenting some ongoing work on the use of isocapacitary estimates and Littlewood-Paley g-functions in establishing functional-embedding type inequalities and describing some connections with the Beurling-Ahlfors operator, Burkholder functional and some profound and longstanding conjectures relating to Morrey'squasiconvexity.