Generalized Fraïssé Constructions and Atomic Models

John T. Baldwin
University of Illinois at Chicago, USA

I will begin with a short survey of extensions of the Fraïssé Construction, pointing out some different directions: Hrushovski Constructions, recent results in neostability/Urysohn spaces, building atomic models in large cardinalities. Then I will discuss:

Theorem 0.0.1 *(Baldwin-Koerwien-Laskowski) There is a family of complete sentences of $L_{\omega_1,\omega}$, ϕ_n for $1 \leq n < \omega$ such that ϕ_n characterizes $\aleph_n$ (has no model of cardinality greater than $\aleph_n$) and all models in $\aleph_n$ are maximal. Each theory satisfies amalgamation in $\aleph_k$ for $k \leq n - 2$, fails it in $\aleph_{r-1}$ and trivially satisfies it in $\aleph_r$. If there is a model of ϕ_n in an uncountable cardinal there are the maximal number.*

I will stress the discovery in this paper of a new notion of n-dimensional amalgamation which allows the construction of *atomic* models in various uncountable cardinals. (Note that the theorem can be thought of as first order model theory by replacing "model of a complete sentence of $L_{\omega_1,\omega}$", by "atomic model of a complete first order theory".)

Background reading:

A field guide to Hrushovski Constructions (with luck I will do some updates)

`http://homepages.math.uic.edu/~jbaldwin/pub/hrutrav.pdf`

Disjoint amalgamation in locally finite AEC

`http://homepages.math.uic.edu/~jbaldwin/pub/cut618.pdf`