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Remarkable Cardinals

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Let us consider the following simple variant of an Ehrenfeucht–Fraïssé game.

I	α		x_0		x_1		$\dots$	
II		λ, β		y_0		y_1		$\dots$

Rules: $\alpha > \kappa$, $\lambda < \beta < \kappa$, $\{x_0, x_1, \dots\} \subset V_\beta$, $\{y_0, y_1, \dots\} \subset V_\alpha$, $x_n \in V_\lambda \Longrightarrow y_n = x_n$, and for every formula φ in the language of set theory and for all $k < \omega$,

$$V_\beta \models \varphi(\lambda, x_0, \dots, x_{k-1}) \Longleftrightarrow V_\alpha \models \varphi(\kappa, y_0, \dots, y_{k-1}).$$

This game is closed, hence determined. A cardinal κ is *remarkable* iff player II has a winning strategy.

Remarkable cardinals are inaccessible, they are a "generic" version of supercompact cardinals, but they are compatible with "$V = L$."

Remarkable cardinals may be used to obtain combinatorial principles at $\aleph_1$ and $\aleph_2$ which, if true at $\kappa > \aleph_2$, yield the existence of $0^\#$. They may also be used to produce the absoluteness of the theory of $L(\mathbb{R})$ with respect to proper forcing, and they measure the strength of Harrington's Principle ("there is a real x such that every x–admissible is an L–cardinal") in 3^{rd} order arithmetic.

Finally, a remarkable cardinal yields a "generic" version of Vopěnka's Principle.

We will develop a few of the key ideas in this area and present proofs of some of the relevant results.

Part of this is joint work with J. Bagaria, V. Gitman, and C. Yong.

Suggested reading:

R. Schindler, *Proper forcing and remarkable cardinals*, Bulletin of Symbolic Logic 6 (2000), pp. 176–184.

R. Schindler, *Proper forcing and remarkable cardinals II*, Journal of Symbolic Logic 66 (2001), pp. 1481–1492.

C. Yong and R. Schindler, *Harrington's principle in higher order arithmetic*, Journal of Symbolic Logic 80 (2015), pp. 477–489.

R. Schindler, *Remarkable cardinals*, in: Infinity, Computability, and Metamathematics (Geschke et al., eds.), Festschrift celebrating the 60th birthdays of Peter Koepke and Philip Welch (2014), pp. 299–308.