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Fraïssé Limits and Homogeneity in Topological Spaces

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The last decade has seen a renewed interest among descriptive set theorists in studying automorphism groups of Fraïssé limits. For example, Kechris, Pestov, and Todorcevic found a fascinating tight connection between the Ramsey property of a Fraïssé family and computation of the universal minimal flow of the automorphism group of the Fraïssé limit of the family.

As it turns out, certain canonical compact topological spaces can be represented as quotients of a version of Fraïssé limits called projective Fraïssé limits. This representation makes it possible to study homeomorphism groups of such compact spaces, which is a continuous context, using discrete arguments and insights from Mathematical Logic. I will outline this approach and show how it is applied in a couple of examples.

For instance, the pseudoarc is the (necessarily unique) generic compact connected subset of the plane, or the Hilbert cube. By a fundamental result of Bing, it is homogeneous as a topological space. By work of Irwin and myself, the pseudoarc is represented as a quotient of a projective Fraïssé limit. In a recent joint work with Tsankov, we determined the correct partial homogeneity of the projective Fraïssé limit associated with the pseudo-arc, which involves combinatorial and basic "dual" model theoretic arguments (e.g., a notion of dual type). Further, we proved a transfer theorem, through which we recover Bing's homogeneity of the pseudo-arc from our partial homogeneity of the projective Fraïssé limit.