The 13th Seminar on
Commutative Algebra and Related Topics, November 16 and 17, 2016
School of Mathematics, IPM, Tehran

Suitable Chains of Semidualizing Modules

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Let R be a commutative Noetherian local ring with maximal ideal $\mathfrak{m}$ and residue field k. A finitely generated R-module C is called semidualizing if the natural homothety map $\chi_C^R : R \longrightarrow \mathrm{Hom}_R(C, C)$ is an isomorphism and $\mathrm{Ext}_R^{>0}(C, C) = 0$. Trivial examples of semidualizing R-modules include R itself and a dualizing R-module when one exists. The set of all isomorphism classes of semidualizing R-modules is denoted by $\mathfrak{G}_0(R)$, and the isomorphism class of a semidualizing R-module C is denoted $[C]$. The set $\mathfrak{G}_0(R)$ has a rich structure, for instance, it comes equipped with an ordering based on the notion total reflexivity. For semidualizing R-modules C and B, we write $[C] \trianglelefteq [B]$ whenever B is totally C-reflexive. A chain in $\mathfrak{G}_0(R)$ is a sequence $[C_n] \trianglelefteq \cdots \trianglelefteq [C_1] \trianglelefteq [C_0]$, and such a chain has length n if $[C_i] \neq [C_j]$ whenever $i \neq j$.

In this talk, we prove that when R is Artinian, the existence of a suitable chain in $\mathfrak{G}_0(R)$ of length $n = \mathrm{Max}\,\{\, i \geqslant 0 \mid \mathfrak{m}^i \neq 0 \,\}$ implies that the the Poincaré series of k and the Bass series of R have very specific forms. Also, in this case we show that the Bass numbers of R are strictly increasing. This gives an insight into the question of Huneke about the Bass numbers of R.

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Some Properties of Higher Iterated Hilbert Coefficients of the Graded Components of Bigraded Modules

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The present paper is motivated by Kodiyalam's work [6], the papers by Theodorescu [11], by Katz and Theodorescu [8], [9] and the paper [3]. In these papers it was shown that for finitely generated R-modules M and N over a Noetherian (local) ring R, and for an ideal $I \subset R$ such that the length of $\mathrm{Tor}_i^R(M, N/I^kN)$ is finite for all k, it follows that the length of $\mathrm{Tor}_R^i(M, N/I^kN)$ is eventually a polynomial function in k. In these papers bounds are given for the degree of these polynomials. In some cases also the leading coefficient is determined. Similar results have been proved for the Ext-modules.

In this paper we consider a related problem. Let $S = K[x_1, \ldots, x_n]$ be the polynomial ring over the field K, and let $I \subset S$ be a graded ideal. Here $I \subset S$ is graded ideal and S is the polynomial ring. It is shown in this paper that for any finitely generated graded S-module M, the modules $\mathrm{Tor}_i^S(M, I^k)$ are finitely graded S-modules which for $k \gg 0$ have constant Krull dimension, and furthermore it is shown that the higher iterated Hilbert coefficients (which appear as the coefficients of the higher iterated Hilbert polynomials) are all polynomial functions. A related result has been shown in [4] for the case M/I^kM and in [5] for the case $\mathrm{Tor}_i^s(S/\mathfrak{m}, I^k)$, where $\mathfrak{m}$ denotes the graded maximal ideal of S.

Observe that knowing all higher iterated Hilbert coefficients of a graded module is equivalent to knowing its h-vector, and hence the Hilbert series of the module. This is the reason why we are not only interested in the ordinary Hilbert coefficients, but in all higher iterated Hilbert coefficients.

The first (and important step) is to show that the higher iterated Hilbert coefficients of the components $A(-a, -b)_k$ of the bi-shifted free A-module $A(-a, -b)$ are polynomial functions in k for $k \gg 0$. This result and the bigraded resolution $\mathbb{F}$ of the bigraded A-module M is then used in the next section to prove the same result for M. There, by using a graded version of the Noether Normalization Theorem, one obtains in this paper upper bounds for the degree of these polynomials. The better bound for the degree of the polynomial function representing the Hilbert coefficient $e_j^i(M_k)$ is achieved when all p_t are the same and it is given by $\deg e_j^i(M_k) \leq \dim M/\mathfrak{m}M + j - 1$. These results are then applied to show that for any finitely generated graded S-module M, and any finitely generated bigraded module

N, the higher iterated Hilbert coefficients of the graded S-modules $\mathrm{Tor}_i^S(M, N_k)$ and $\mathrm{Ext}_s^i(M, N_k)$ are polynomial functions in k for $k \gg 0$.

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Maximal Subrings of Affine Rings

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We prove that if $R = F[\alpha_1, \ldots, \alpha_n]$ is an affine integral domain over a field F, then R has only finitely many maximal subrings if and only if F has only finitely many maximal subrings and each α_i is algebraic over F, which is similar to the celebrated Zariski's Lemma.

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Model Structures on the Category of Complexes of Quiver Representations

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Model categories, first introduced by Quillen in 1967, form the foundation of homotopy theory. Model categories are used to give an effective construction of the localization of categories, where the problem is to convert a class of morphisms called weak-equivalences into isomorphisms. Because this idea of inverting weak equivalences is so central in mathematics, model categories are extremely important.

In this talk, we study the category $\mathbb{C}(\mathrm{Rep}(\mathcal{Q}, \mathcal{G}))$ of complexes of representations of quiver $\mathcal{Q}$ with values in a Grothendieck category $\mathcal{G}$. We develop a method for constructing some model structures on $\mathbb{C}(\mathrm{Rep}(\mathcal{Q}, \mathcal{G}))$ based on componentwise notion. As an application of these model structures we introduce some descriptions of the derived category of complexes of representations of $\mathcal{Q}$ in Mod-R. In particular we set $\mathcal{Q} = A_2$ and consider the morphism category $\mathbf{H}(R)$ and its two full subcategories, monomorphism category $\mathbf{S}(R)$ and epimorphism category $\mathbf{F}(R)$. We show that the well know equivalence between $\mathbf{S}(R)$ and $\mathbf{F}(R)$ can be extended to an auto-equivalence of derived category of $\mathbf{H}(R)$.

This talk is based on a joint work with R. Hafezi.

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Characterizations of Commutative Rings by their Simple, Cyclic, Uniform and Uniserial Modules

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The subject of determining structure of rings and algebras over which all modules are direct sums of certain modules (such as simple, cyclic, unuserial, uniform or indecomposible modules) has a long history. One of the first important contributions in this direction is due to Wedderburn and Artin. They showed that every module over a semisimple algebra R is a direct sum of simple modules if and only if R is isomorphic to a direct product of matrix rings over division rings. Another one is due to G. Köthe. He considered rings over which all modules are direct sums of cyclic modules and proved that over an Artinian principal ideal ring, each module is a direct sum of cyclic modules. Later I. S. Cohen and I. Kaplansky obtained that if R is a commutative ring such that each R-module is a direct sum of cyclic modules, then R must be an Artinian principal ideal ring. Rings, over which all modules are serial, were first systematically consider by T. Nakayama. He proved that if R is an Artinian serial ring, then every R-module is serial. The converse is obtained by L. A. Skornyakov. Thus a ring R is Artinian serial if and only if every R-modules are serial.

Now, some natural problems arise from this situation. Instead of considering rings for which all modules are direct sums of simple, cyclic, unuserial, uniform or indecomposible modules, we weaken these conditions and study the structures of rings R for which it is assumed only that the ideals (or proper ideals) of R are direct sums of such modules. For instance, we will discuss the following natural questions in the commutative cases:
(i) Which rings have the property that every ideal is a direct sum of cyclic modules?
(ii) Which rings have the property that every proper ideal is a direct sum of virtuallly simple modules?
(iii) Which rings have the property that every proper ideal is serial?

Also, we introduced and study *prime serial modules* and *almost serial modules* as two generalizations of serial modules.

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Annihilator of Local Cohomology Modules

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One classical topic in the study of local cohomology is whether the non-vanishing of a specific local cohomology module is equivalent to the vanishing of its annihilator; this has been studied by several authors, including Huneke, Koh, Lyubeznik and Schenzel. Motivated by this, we give a review of the annihilator of local cohomology modules from different points of view with a road map of recent work to open questions.

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Involutive Bases and its Applications

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Gröbner bases and the first algorithm to compute them were introduced by Bruno Buchberger is his PhD thesis in 1965. Since then, the theoretical and computational issues related to Gröbner bases have been extensively developed and Gröbner bases have become a powerful and a widely used tool in polynomial ideal theory. Gröbner bases have applications to many problems such as ideal membership problem, solving polynomial systems, geometric theorem proving, syzygy module and so on. It is well-known that a Gröbner basis allows to compute important invariants of the ideal it generates such as dimension, degree and Hilbert function. However, it does not inherit, in general, further algebraic and combinatorial properties of the ideal such as satiety, Castelnuovo-Mumford regularity, depth and Cohen-Macaulayness. Therefore, involutive bases were introduced, as a special kind of Gröbner bases, with additional properties. The origin of these bases lies in the Janet-Riquier theory of linear systems of partial differential equations. In this direction, based on the methods described in the book by Pommaret, Zharkov and Blinkov in 1996 introduced the concept of involutive polynomial bases. Later on, Gerdt, Blinkov, Zharkov, and others studied the notions of involutive division and involutive bases. In this talk, we review first the theories of Gröbner bases and involutive bases and discuss some applications of these concepts in commutative algebra and algebraic geometry.

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Monomial Curves of Homogeneous Type

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Let S be a numerical semigroup generated by $n_1, \ldots, n_d$, $\Bbbk$ be a field and $\Bbbk[S] = \Bbbk[t^{n_1}, \ldots, t^{n_d}] \subseteq \Bbbk[t]$ be the associated semigroup ring. The kernel of the map $\varphi : \Bbbk[x_1, \ldots, x_d] \longrightarrow \Bbbk[S]$ defined as $\varphi(x_i) = t^{n_i}$, is indeed the defining ideal of the monomial curve parametrized by $x_1 = t^{n_1}, \ldots, x_d = t^{n_d}$ and is denoted by $I(S)$. By definition, each element $s \in S$ can be written as $s = \sum_{i=1}^{d} r_i n_i$ for some non-negative integers r_i. We consider the order of s as the integer k such that $s \in kM \setminus (k+1)M$. In particular s may be represented as $s = r_1 n_1 + \cdots + r_d n_d$ $(r_i \geq 0)$ with $\sum_{i=1}^{d} r_i = k$ maximum over all expressions of s in S, which we call it a maximal expression of s.

Let $G(S)$ denote the coordinate ring of the tangent cone of $\Bbbk[S]$, which is precisely the associated graded ring $\mathrm{gr}_{\mathfrak{m}}(\Bbbk[S])$ with respect to the maximal ideal $\mathfrak{m} = (t^{n_1}, \ldots, t^{n_d})$. By a general result due to Robbiano (see [6], [2]), Betti numbers of the associated graded ring $G(S)$ are above bounds for Betti numbers of the semigroup ring R i.e. $\beta_i(\Bbbk[S]) \leq \beta_i(G(S))$ for all $i \geq 1$. The semigroup S is called *of homogeneous type* if $\beta_i(\Bbbk[S]) = \beta_i(G(S))$ for all $i \geq 1$. Since $\Bbbk[S]$ is one dimensional Cohen-Macaulay rings, if S is of homoheneous type, then $G(S)$ is Cohen-Macaulay.

In this talk, we introduce the concept of homogeneous numerical semigroups and show that all homogeneous numerical semigroups with Cohen-Macaulay tangent cones are of homogeneous type.

Recall that, for an element $s \in S$, the Apéry set of S with respect to s is defined as $\mathrm{AP}(S, s) = \{x \in S \mid x - s \notin S\}$. We say that S is *homogeneous*, when all expressions of elements in $\mathrm{AP}(S, a_1)$ are maximal. As examples of homogeneous numerical semigroups, one may consider numerical semigroups generated by generalized arithmetic sequences, Frobenius numerical semigroups, those of embedding dimension 2 and those of (almost) maximal embedding dimensions. We may also show that the homogeneous property of S and Cohen-Macaulay property of $G(S)$ preserve under special gluings and so we are able to construct a large family of homogeneous numerical semigroups with Cohen-Macaulay tangent cones.

If $G(S)$ is complete intersection, then S is also a complete intersection and both S and $G(S)$ have the same number of minimal generators. Hence S is of homogeneous type. In embedding dimension three, we classify all numerical semigroups of homogeneous type showing that a numerical semigroup S with embedding dimension three is of homogeneous type precisely when, $G(S)$ is complete intersection or S is homogeneous with Cohen-Macaulay tangent cone. We also investigate numerical semigroups of homogeneous type of embedding dimension four, in some special cases.

J. Herzog and H. Srinivasan conjectured that for $j \gg 0$ the Betti numbers of the ideals $I(\mathbf{n}+j)$ become periodic on j with period $n_d - n_1$. In 2013, the conjecture was proven to be true by A. V. Jayanthan and H. Srinivasan for $d = 3$ [4], by A. Marzullo for some particular cases if $d = 4$ [5], and by P. Gimenez, I. Senegupta and H. Srinivasan in the case of arithmetic sequences [1]. Finally, in 2014, T. Vu gave a completely general positive answer in [7]. One of the main ingredients of Vu's proof is that there exists a positive integer N such that any minimal binomial non-homogeneous generator of $I(\mathbf{n}+j)$ is of the form $x_1^{a_1}u - vx_d^{a_d}$, where a_1, a_d are positive integres, u, v are monomials in the variables $x_2, \ldots, x_{d-1}$, and $\deg x_1^{a_1}u > \deg vx_d^{a_d}$. It is noteworthy that the bound N depends on the Castelnuovo-Mumford regularity of $J(\mathbf{n})$, the ideal generated by the homogeneous elements in $I(\mathbf{n})$. In addition, we prove that there exists a positive integer L such that for any $j > L$, all the numerical semigroups generated by sequences of the form $\mathbf{n}+j$ are homogeneous and have Cohen-Macaulay tangent cone, so in particular they are of homogeneous type. The novelty here is that the constant L only depends on the sequence of integers $\mathbf{n}$ and can be easily computed. In fact, it can be computed in terms of what we call the shifting type of a numerical semigroup : two numerical semigroups can be obtained one from another as a shifting if and only if they have the same shifting type. So our results say that in the class of numerical semigroups with the same shifting type, all numerical semigroups except a finite number, that only depends of its shifting type, are of homogeneous type.

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Asymptotic Grade and Analytic Spread

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Let I be an ideal of a commutative Noetherian ring R, and let $\bar{I}$ denote the integral closure of I. Ratliff showed that the sequence of associated prime ideals

$$\mathrm{Ass}_R\, R/\overline{I^n}, n = 1, 2, \ldots,$$

becomes eventually constant, the stable value being denoted by $\overline{A^*}(I)$. In [2], Rees defined an asymptotic sequence over an ideal, as follows:

Definition 1 *Let I be an ideal of a Noetherian ring R. A sequence $\mathbf{x} = x_1, \ldots, x_n$ of elements of R is called an asymptotic sequence over I if, $(I, (\mathbf{x}))R \neq R$ and for all $1 \leq i \leq n$, we have $x_i \notin \bigcup\{\mathfrak{p} \in \overline{A^*}((I, (x_1, \ldots, x_{i-1}))R)\}$.*

Let J be a second ideal of R, in [1], we showed that all maximal asymptotic sequences over I coming from J have the same length, which we denote by $\mathrm{agd}_I(J)$. It follows from [1, Theorem 4.6] that if R is local, then

$$\mathrm{agd}_I(J) \leq \ell(J);$$

here $\ell(J)$ is the analytic spread of J. In this talk we give some necessary and sufficient conditions for equality to hold. Before we state the main result, recall that J is said to be projectively equivalent to I in case $\overline{I^i} = \overline{J^j}$ for some positive integers i, j, and J is said to be a reduction of I in case $J \subseteq I$ and $JI^n = I^{n+1}$ for some non-negative integer n. Further, J is a minimal reduction of I, if no ideal strictly contained in J is a reduction of I.

Theorem 1 *Let I and J be ideals of a Noetherian local ring $(R, \mathfrak{m})$. Then the following are equivalent:*

(i) $\mathrm{agd}_I(J) = \ell(J)$,

(ii) *There exists an ideal which is projectively equivalent to J and is generated by an asymptotic sequence over I of length $\ell(J)$,*

(iii) *If K is an ideal that is projectively equivalent to J and is generated by $\ell(J)$ elements, then every minimal basis of K is an asymptotic sequence over I of length $\ell(J)$.*

Also, if $R/\mathfrak{m}$ is infinite, the above are equivalent to:

(iv) *There exists a reduction of J which is generated by an asymptotic sequence over I,*

(v) *If K is an ideal that is projectively equivalent to J, then every minimal basis of each minimal reduction of K is an asymptotic sequence over I.*

Finally, we give several consequences of these characterizations. For instance, we give four descriptions of $\overline{A^*}(I+J)$, for all ideals I and J with $\operatorname{agd}_I(J) = \ell(I+J)$ as follows:

Corollary 1 *Let I and J be ideals of a Noetherian local ring $(R, \mathfrak{m})$ such that $\operatorname{agd}_I(J) = \ell(I+J)$, then the following are equal:*

(i) $\overline{A^*}(I+J)$,

(ii) $\{\mathfrak{p} \in V(I+J) \mid \ell(I+J) = \operatorname{agd}_{IR_\mathfrak{p}}(\mathfrak{p}R_\mathfrak{p})\}$,

(iii) $\{\mathfrak{p} \in V(I+J) \mid \ell(I+J) = \dim R^*_\mathfrak{p}/z - \ell((IR^*_\mathfrak{p} + z)/z)$ for some minimal prime z in $R^*_\mathfrak{p}\}$,

(iv) $\{\mathfrak{p} \in V(I+J) \mid \ell(I+J) = \dim R^*_\mathfrak{p}/z$ for some minimal prime z in $R^*_\mathfrak{p}\}$,

(v) $\{\mathfrak{p} \in V(I+J) \mid \ell(I+J) = \operatorname{agd}_{(0)R_\mathfrak{p}}(\mathfrak{p}R_\mathfrak{p})\}$.

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On the Existence of Certain Modules of Finite Complete Intersection Homological Dimensions

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In this lecture, we will use complete intersection homological dimensions for characterizing local rings which are either regular, complete intersection, Gorenstein or Cohen-Macaulay. Let $(R, \mathfrak{m}, k)$ be a commutative Noetherian local ring. We show that R is Cohen-Macaulay if there exists either a nonzero Cohen-Macaulay R-module of finite complete intersection projective dimension or a nonzero finitely generated R-module of finite complete intersection injective dimension. Also, we prove that R is Gorenstein if and only if there exists a nonzero R-module M with finite depth such that its injective dimension and its complete intersection projective dimension are finite. In addition, we prove that R is complete intersection if and only if the complete intersection injective dimension of k is finite.

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Hironaka Group Schemes in Positive Characteristic

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Group schemes play a central role in the study of the blow up of a scheme along a regular center. This was pointed out by Hironaka for the case in which the scheme is normally flat along such center, and he proves that resolution of singularities can be achieved by blowing up successively along normally flat centers when the characteristic is zero. Groups schemes arise in the study of this blow up when the characteristic is non-zero.

Let k be a field of characteristic $p > 0$. In Connection with resolution of singularity in characteristic p, Hironaka associated an additive group scheme $B_{\mathbb{P}_n,\mathfrak{p}}$ to a point $\mathfrak{p}$ of projective space $\mathbb{P}_n$ over k as follows: Let $S = k[X_0, \dots, X_n]$ be a polynomial ring over a field k and $\mathfrak{p} \in \mathbb{P}_n = \mathrm{Proj}(S)$. Let $I_\mathfrak{p}$ be the ideal of S generated by forms H such that the multiplicity of $\mathrm{Proj}(S/H) = \deg H$ at point $\mathfrak{p}$. By the main result of Hironaka [1], $I_\mathfrak{p}$ is generated by additive homogeneous polynomials. i.e. elements of the form $a_0X_0^q + a_1X_1^q + \dots + a_nX_n^q$ with a_i in k and q a power of the characteristic p. Thus $B_{\mathbb{P}_n,\mathfrak{p}} = \mathrm{Spec}(S/I_\mathfrak{p})$ is a subgroup scheme of the vector group scheme $\mathrm{Spec}(S)$. We call $B_{\mathbb{P}_n,\mathfrak{p}}$ Hironaka group scheme associated to $\mathfrak{p}$.

The aim of this talk is to discuss the structure of Hironaka group scheme. Also we study the behavior of Hironaka group schemes under field extensions. This talk is based on a joint work with Orlando Villamayor.

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Cotangent Cohomology of Quadratic Monomial Ideals

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In deformation theory of affine schemes there is a cohomology theory which assigns to any $\Bbbk$-algebra A two cohomology modules called the first and second cotangent cohomology modules denoted by $T^1(A)$ and $T^2(A)$. The first cotangent cohomology module characterizes the first order deformations of A and the second cotangent cohomology module contains the obstructions for lifting these deformations. In this talk we investigate cotangent cohomology of quotients of polynomial rings by quadratic monomial ideals.

Deformation theory of square-free monomial ideals have been studied by Klaus Altmann and Jan Arthur Christophersen in [3, 2]. If I is a square-free monomial ideal in a polynomial ring $R = \Bbbk[x_1, \ldots, x_n]$ then $T^1(R/I)$ is $\mathbb{Z}^n$-graded. In [3, 2] the authors give a combinatorial description of each $\mathbb{Z}^n$-graded part of $T^1(R/I)$. This description of the first cotangent cohomology is essentially used in [1] to classify rigid square-free monomial ideals and in particular find the class of rigid edge ideals of graphs.

Any ideal in a polynomial ring with a Gröbner basis consisting of quadrics degenerate to a quadratic monomial ideal. Such ideals include Hibi ideals and Plücker relations of Grassmann varieties.

A quadratic monomial ideal I in a polynomial ring $R = \Bbbk[x_1, \ldots, x_n]$ gives rise to a (not necessarily simple) graph $G = (V(G), E(G))$ where $V(G) = \{x_1, \ldots, x_n\}$ and $E(G) = \{\{x_i, x_j\} \mid x_i x_j \in I\}$. We use the combinatorics of the corresponding graph to describe a generating set for the first cotangent cohomology module of the ring R/I as well as vanishing results for the second cotangent cohomology module.

Due to simplicity of relations of a quadratic monomial ideal it is possible to apply a more direct approach towards description of the first cotangent cohomology module. Here we construct a homogeneous generating set for $T^1(R/I)$ as a $\mathbb{Z}$-graded module. This generating set is easier to use when examining the rigidity of a quadratic monomial ideal.

The second cotangent cohomology module is an obstruction space and only its vanishing is of importance. Characterization of quadratic monomial ideals for which the second cotangent cohomology vanishes seems to be difficult in general. However when G is a simple graph with no 3-cycles then there is a nice characterization for vanishing of the second cotangent cohomology module. We also show that if the graph G does not have any induced 3 or 4 cycles then the second cotangent cohomology module vanishes.

This talk is based on the paper [4].

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Some Topics in Derived Local Cohomology

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Let Z be a specialization closed subset of $SpecR$ and X a complex in the derived category $D(R)$. In this talk, we briefly review some recent results concerning the notion of derived local cohomology modules $H^i_Z(X)$. First, we show that all of the previously known generalizations of local cohomology such as generalized local cohomology with respect to two modules, local cohomology with respect to a pair of ideals and abstract local cohomology are special cases of the notion $H^i_Z(X)$. Next, we consider the derived category analogues of some important results in the theory of local cohomology such as Faltings' Local-global Principle, Faltings' Annihilator and the Hartshorne-Lichtenbaum Vanishing Theorems.

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Symmetry in Vanishing of Tate Cohomology over Gorenstein Rings

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We investigate symmetry in the vanishing of Tate cohomology for finitely generated modules over local Gorenstein rings. For finitely generated R–modules M and N over Gorenstein local ring R, it is shown that $\widehat{\operatorname{Ext}}^i_R(M,N) = 0$ for all $i \in \mathbb{Z}$ if and only if $\widehat{\operatorname{Ext}}^i_R(N,M) = 0$ for all $i \in \mathbb{Z}$.

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Multiplicative Ideal Theory in Graded Domains

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In this talk we give some graded ideal theoretic properties of graded domains. Let $R = \bigoplus_{\alpha\in\Gamma} R_\alpha$ be a graded integral domain and $\star$ be a semistar operation on R. For $a \in R$, denote by $C(a)$ the ideal of R generated by homogeneous components of a and for$f = f_0 + f_1X + \cdots + f_nX^n \in R[X]$, let $\mathcal{A}_f := \sum_{i=0}^{n} C(f_i)$. Let $N(\star) := \{f \in R[X] \mid f \neq 0$ and $\mathcal{A}_f^\star = R^\star\}$. In this talk we study relationships between ideal theoretic properties of $\mathrm{NA}(R,\star) := R[X]_{N(\star)}$ and the homogeneous ideal theoretic properties of R.

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Secondary Representation in Abelian Categories

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We want to talk about the secondarey representation theory of abelian categories. Let$\mathcal{A}$ be a Grothendieck category. In this lecture we define attached atoms of nonzero objects of $\mathcal{A}$ as a dual of the associated atoms of nonzero objects introduced by Kanda [K1]. If R is a (non)commutative ring, then we show that this new notion can be corresponded to the attached prime ideals of nonzero modules introduced by Macdonald [M] and Annin [A].

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On the Minimal Free Resolution of Graded Ideals

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In this talk we study minimal free resolutions of ideals by the mapping cone technique. we give some sufficient conditions to check the minimality of the resolution obtained by mapping cone. Then we introduce and study some classes of ideals whose minimal free resolution can be obtained by iterated mapping cone. In particular, we discuss about resolutions of ideals with $\{d_i\}$-linear quotients, ideals with $\{t_i\}$-regular quotients and ideals of the form $J_{\mathcal{H}} + (x_{i_1}^2, \ldots, x_{i_m}^2)$ where $J_{\mathcal{H}}$ is the edge ideal of a hypergraph $\mathcal{H}$.

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Linear Resolution in Non-zero Characteristic

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Let I be an ideal generated by square-free monomials of degree d in a polynomial ring over a field K. A clutter C can be corresponded to I in a natural way. The aim of this talk is to discuss about combinatorial behavior of the clutter such that the corresponding ideal gets a linear resolution over a field of a prime characteristic. We present some examples which have a linear resolution over any field except fields of characteristic p for a given prime p.