Jacobian linear type hypersurfaces with isolated singularities

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Abstract

Let $R = k[x_1, \dots, x_n]$ be a polynomial ring over an algebraically closed field k of characteristic zero. Let $X = V(f)$ be a reduced singular hypersurfaces with isolated singularities which defined by a reduced polynomial $f \in R$. The singular locus of X is defined by the Jacobian ideal $I_f = (f, J_f)$, where $J_f = (\partial f/\partial x_1, \dots, \partial f/\partial x_n)$ is so called the gradient ideal. We say that the hypersurface X is of *Jacobian linear type* if the Jacobian ideal $I_f \in R$ is of linear type. In general, an ideal I in a commutative ring A is called of linear type if the symmetric algebra of I is isomorphic with the Rees algebra.

The principal question motivated this lecture is: which hypersurfaces are of Jacobian linear type? An elegant answer to this question is very important in algebraic geometry. Actually, if the hypersurface X is of Jacobian linear type then for computing characteristic cycles of a hypersurface in intersection theory we need just compute cycles of the naive blow-up. In this talk, we show that an affine hypersurface X is of Jacobian linear type if and only of X is locally Eulerian. In the sequel, we prove that a projective hypersurface is of gradient linear type if and only if the corresponding affine hypersurface in the affine chart associated to singular point is locally Eulerian. In particular, we prove that Nodal and Cuspidal projective plane curves are of gradient linear type.