Abstracts

(in alphabetic order)

Workshop (December 6-8, 2016)

Joan Bosa (University of Glasgow, UK)

The Cuntz semigroup: Classification of C-algebras*

Abstract: Over the last 25 years, the classification of simple, nuclear C*-algebras has inspired a great wealth of research. Recently, a classification has been carried out in two ground-breaking articles [5, 7] for simple C*-algebras of finite nuclear dimension. Nuclear dimension plays the role of a non-commutative covering dimension for nuclear C*-algebras. Requesting this dimension to be finite is one of the strong regularity conditions occurring in the Toms-Winter conjecture, which predicts that three regularity conditions, each with a different flavour, are in fact equivalent. Counterexamples to the conjecture stating that the same classifying invariant, which works in the case of finite nuclear dimension (the so-called Elliott invariant), should work in the general case, appeared in 2003 due to Rørdam [6] and in 2008 due to Toms [8]. The latter exhibited two non-isomorphic AH-algebras that agreed not only on the Elliott invariant, but also on a whole swathe of topological invariants.

However, Toms' examples can be distinguished using the Cuntz semigroup Cu( ). The Cuntz semigroup has became a prominent tool in classification of C*-algebras since then, and, in broad outline, the purpose of this course is twofold. On the one hand, we will provide an account on the basics of the Cuntz semigroup showing for instances the equivalent definitions of it, the definition of the abstract category Cu and the important result obtained by [2], which shows that the Cuntz semigroup can be seen as a continuous functor from (stable) C*-algebras to the new category Cu. On the other hand, we shall survey some of the most prominent applications of this invariant to the classification programme such as its relation with K-theory and traces for a wide class of C*-algebras ([4]), and the recent definition of the bivariant theory for the Cuntz semgiroup [3].

We encourage the reader to look at the references displayed in this abstract, and check [1] for further details.

1. P. Ara, F. Perera, A. S. Toms; K-theory for operator algebras. Classification of C*-algebras. Aspects of operator algebras and applications, 1–71, Contemp. Math., 534, Amer. Math. Soc., Providence, RI, 2011.

2. K. T. Coward, G. A. Elliott, C. Ivanescu; The Cuntz semigroup as an invariant for C*-algebras, J. Reine. Angew. Math. 623 (2008), 161–193.

3. J. Bosa, G. Tornetta, J. Zacharias; A bivariant theory for the Cuntz semigroup, preprint, arXiv:1602.02043.

4. N. P. Brown, F. Perera, A. Toms; The Cuntz semigroup, the Elliott conjecture and dimension functions on C*-algebras, J. Reine. Angew. Math. 621 (2008), 191–211 .

5. G. A. Elliott, G. Gong, H. Lin, Z. Niu; On the classi_cation of simple amenable C*-algebras
with _nite decomposition rank, II, preprint. ArXiv:1507.03437v2.

6. M. R_rdam; A simple C*-algebra with a _nite and an in_nite projection, Acta Math. 191
(2003), 109{142.

7. A. Tikuisis, S. White, W. Winter; Quasidiagonality of nuclear C*-algebras, preprint. ArXiv:
1509.08318.

8. A. S. Toms; On the classi_cation problem for nuclear C*-algebras, Ann. of Math. 167 (2008),
1029{1044.

Nate Brown (Penn State University, USA)

Nuclear C-algebras*

Abstract: In the 1970's Alain Connes made stunning breakthroughs in the theory of simple injective von Neumann algebras. The structure of these ubiquitous algebras was laid bare, leading to a complete classification and applications in other areas such as dynamical systems. The C*-analogue of injectivity is nuclearity, a small army of researchers have been working for decades to understand the structure of this fundamental class of C*-algebras. And we're almost there, due to some breathtaking recent advances. I will survey the history and highlights of this remarkable body of work. It's a long story, with lots of players, but it's a good one.

Marzieh Forough (IPM, Iran)

Traces on crossed product of C-algebras*

Abstract: Group actions on C*-algebras and their crossed products is one of the most central subjects in operator algebras. In particular, permanence properties for crossed products by actions of finite groups have been extensively studied. In this talk, I discuss the stability of some properties of C*-algebras under taking crossed products by actions of finite groups with Rokhlin property or tracial Rokhlin property.

In particular, I talk about the behavior of the finite dimensional approximation properties of traces on these crossed products, when all traces on the original C*-algebra are uniformly quasidiagonal or uniformly locally finite dimensional.

Nasser Golestani (Tarbiat Modares University & IPM, Iran)

A categorical *approach to classification of C*-algebras*

Abstract: A categorical formulation of the Elliott's conjecture is that the K-theory functor from a category of relatively nice (i.e., separable simple nuclear) C*-algebras to the category of Elliott invariants is a (strong) classification functor. A theorem of Elliott in his Advances paper in 2010 says that the functor from the category of all separable C*-algebras to an abstract category is a strong classification functor. We discuss the idea of using this theorem to find some results in the Elliott's program. In particular, we show that for AF algebras, the abstract category is equivalent to the (concrete) categories of dimension groups and Bratteli diagrams. We hope to extend these results to the classification of Lin's TAF algebras using some results of Dadarlat.

The talk is based on a joint work with Massoud Amini and George A. Elliot.

Jianchao Wu (Penn State University, USA)

Colors and towers: Noncommutative dimensions and dynamics

Abstract: Developments in the classification and the structure theory of C*-algebras in the past decade have highlighted the importance of an assortment of regularity properties, one of the most prominent of them being the property of having finite nuclear dimension. This has spurred growing interests in the advances of noncommutative dimension theories, for which a focal challenge is to find ways to control noncommutative dimensions for crossed product C*-algebras. To this end, various dimensions of dynamical nature have been developed, including Rokhlin dimension, dynamical asymptotic dimension, amenability dimension, etc. Roughly speaking, these dimensions measure the complexity of the topological or C*-dynamical system that gives rise to a given crossed product. We will discuss some of these concepts as well as their applications.

Mini Courses (December 3 and 4, 2016)

Marzieh Forough (IPM, Iran)

Nuclear and Rokhlin dimensions

Abstract: I first review some basic facts concerning nuclear dimension and decomposition rank for nuclear C*-algebras. Then I will talk about the notion of Rokhlin dimension for actions of finite groups and group of integers. In particular, some permanence properties of crossed products by actions with finite Rokhlin dimension will be discussed.

[1] I. Hirshberg, W. Winter, J. Zacharias, Rokhlin dimension for C*-dynamical systems. Comm. Math. Phys. 335)2015), 637-670.
[2] W. Winter, J. Zacharias, The nuclear dimension for C*-algebras. Adv. Math. 222. (2002), 461-498
[3] E. Kirchberg, W. Winter, Covering dimension and quasidiagonality. Internat. Math. 15 (2004), 63-85.

Nasser Golestani (Tarbiat Modares University & IPM, Iran)

Lecture 1: Classification of C-algebras via Elliott invariant*
Lecture 2: Cuntz semigroup

Abstract: The classification program of George A. Elliott is now 4o years old. An approximately finite-dimensional (AF) C*-algebra is a C*-algebra that is the inductive limit of a sequence of finite-dimensional C*-algebras. These algebras were first described by the pioneering works of Glimm [2] and Bratelli [1]. Elliott [3] gave a complete classification of AF algebras using the K0 functor (some nice ordered abelian groups).

It is proposed by Elliott that other classes of C*-algebras are classifiable by K-theoretic invariants. The Elliott invariant consists of K-theories (and order structure), the scale (the representatives of the projections in the algebra), tracial states, and a natural pairing between them and K0-group. Another important invariant is the Cuntz semigroup (equivalence classes of positive elements in the stabilization of the algebra).

We give basic facts about classification program and the Elliott invariant and Cuntz semigroup.

[1] O. Bratteli, Inductive limits of finite dimensional C*-algebras, Trans. Amer. Math. Soc. 171 (1972), 195-234.
[2] J. Glimm, On a certain class of operator algebras, Trans. Amer. Math. Soc. , 95 (1960) 318–340.
[3] G.A. Elliott, On the Classification of inductive limits of sequences of semisimple finite-dimensional algebras, J. Algebra 38 (1976), 29-44.