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The Green-Lazarsfeld Index of Veronese Embeddings

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In order to attain better understanding of a projective variety classically there is a great interest to determine the equations of a projective variety and also the syzygies of its homogeneous ideal. In this regard, M. Green and R. Lazarsfeld defined the property N_p which, roughly speaking, refers to the simplicity of syzygies of the homogeneous coordinate ring of a smooth projective variety embedded by a very ample line bundle. In the case $X = \mathbb{P}^n$ with the line bundle $\mathcal{O}_{\mathbb{P}^n}(d)$, M. Green proved that the coordinate ring of the image of Veronese embedding of degree d of X satisfies the property N_d . W. Bruns, A. Conca and T. Romer improved the lower bound of the Green-Lazarsfeld index of the Veronese subring $S(d)$ to $d + 1$. Their approach is based on investigation of the homological invariants of the Koszul cycles and Koszul homologies of the d -th power of the maximal ideal. By aforementioned motivation A. Conca and S. Murai studied the Castelnuovo-Mumford regularity of the Koszul cycles $Z_t(I, S)$ of a homogeneous ideal in a polynomial ring S .

In this talk, I will present my recent results which extend to one-dimensional quotients the result of A. Conca and S. Murai on the convexity of the regularity of Koszul cycles. By providing a relation between the regularity of Koszul cycles and Koszul homologies, we prove a sharp bound for the regularity of the Koszul homologies of a homogeneous ideal in a polynomial ring.