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Atom Supports and Associated Atoms in Abelian Categories

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Throughout this talk, $\mathcal{A}$ is an abelian category and all subcategories of $\mathcal{A}$ are full and closed under isomorphisms. In this talk, we study monoform objects and atoms introduced by Kanda. We define atomical decomposition of any subobject of a noetherian object M of $\mathcal{A}$ and show that if M is a noetherian object, then every subobject N has an atomical decomposition which determines the associated atoms of M/N. We define maximal atoms and present sufficient conditions, so that an object has a maximal atom in its atom support. We also show that the maximal atoms can characterize finite length objects.

We study the relation between monoform objects and full subcategories of an abelian category. We show that if $\mathcal{A}$ is a locally noetherian Grothendieck category, there is a bijection between full subcategories of $\mathcal{A}$ which are closed under subobjects, injective envelopes and direct sums, and subclasses of $\mathrm{ASpec}(\mathcal{A})$.

In sequel we study the localization theory in terms of atoms due to Kanda [1]. We show that several results in the classical localization theory of module category can be true in Grothendieck categories in sense of the new version of localization. More precisely, we prove that if α is an atom, then $\mathrm{E}(\alpha)_\alpha = \mathrm{E}(\alpha)$, where $\mathrm{E}(\alpha)$ is the injective envelope of α. We also show that if $\mathcal{X}$ is a localizing subcategory of $\mathcal{A}$ with the canonical functor $F : \mathcal{A} \to \mathcal{A}/\mathcal{X}$ and the idempotent radical $t_{\mathcal{X}}$ and also if M is an object of $\mathcal{A}$, then $\mathrm{AAss}\, F(M) = \mathrm{AAss}(M/t_{\mathcal{X}}(M))$. Moreover if $\mathcal{X}$ is stable, then $\mathrm{AAss}\, F(M) = \mathrm{AAss}(M) \setminus \mathrm{ASupp}\, \mathcal{X}$. We prove that if N is a proper subobject of a noetherian object M in $\mathcal{A}$ and α is a minimal atom in $\mathrm{ASupp}(M/N)$ such that $\mathcal{X}_\alpha$ is stable, then the α-component of any atomical decomposition of N is $\eta_M^{-1}(GF(N))$ where $F : \mathcal{A} \to \mathcal{A}_\alpha$ is the canonical functor with the right adjoint functor $G : \mathcal{A}_\alpha \to \mathcal{A}$. This fact implies that the α-component is uniquely determined only by M, N and α.

We end our talk by giving an applications of our results the fully right bounded noetherian rings.

References

[1] R. Kanda, *Specialization orders on atom specta of Grothendieck categories*, J. Pure Appl. Algebra. 219 (2015), no. 11, 4907–4952.