Abstracts of the Seminar Talks

(In Alphabetical Order)

The 15th Seminar on
Commutative Algebra and Related Topics, January 16 and 17, 2019
School of Mathematics, IPM, Tehran

The Absolutely Koszul and Backelin-Roos Properties for Spaces of Quadrics of Small Codimension

Rasoul Ahangari Maleki
IPM, Iran

Let $\boldsymbol{k}$ be a field and let R be a standard graded quadratic $\boldsymbol{k}$-algebra with $\dim_{\boldsymbol{k}} R_2 \leq 3$. We construct a graded surjective Golod homomorphism $\varphi\colon P \to R$ such that P is a complete intersection of codimension at most 3. Furthermore, we show that R is absolutely Koszul (that is, every finitely generated R-module has finite linearity defect) if and only if R is Koszul if and only if R is not a trivial fiber extension of a standard graded $\boldsymbol{k}$-algebra with Hilbert series $(1+2t-2t^3)(1-t)^{-1}$. In particular, we recover earlier results on the Koszul property of Backelin [1], Conca [3] and D'Alì [5].

This talk is based on a joint work with Liana M. Şega.

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Fully Decomposability of Balanced Big Cohen-Macaulay Modules

Abdolnaser Bahlekeh
Gonbade-Kavous University, Iran

A famous result of Auslander [1] (also Ringel-Tachikawa [5]) asserts that an artin algebra Λ is of finite representation type if and only if every left Λ-module is fully decomposable, i.e., it is a direct sum of finitely generated modules. Recall that an artin algebra Λ is of finite representation type, provided that the set of isomorphism classes of indecomposable finitely generated modules is finite. Motivated by Auslander's result, studying decomposition of Gorenstein projective modules over artin algebras into finitely generated ones has been the subject of several expositions. In particular, Chen [4] has proved that a Gorenstein artin algebra Λ is of finite Gorenstein representation type, in the sense that there are only finitely many isomorphism classes of indecomposable finitely generated Gorenstein projective Λ-modules, if and only if any left Gorenstein projective Λ-module is fully decomposable. This result has been extended to virtually Gorenstein artin algebras by Beligiannis [3]. In this talk, which is based on a joint work with Shokrollah Salarian and Fahimeh Sadat Fotouhi, we will discuss on fully decomposability of balanced big Cohen–Macaulay modules over a complete Cohen–Macaulay local ring. Precisely, the major issues which will be considered in this talk are, when a given balanced big Cohen–Macaulay module is fully decomposable; when *every* balanced big Cohen–Macaulay module is so; analogues of the first Brauer-Thrall conjecture for modules and analogues result for Gorenstein projective modules over Cohen–Macaulay artin algebras in the sense of Auslander and Reiten [2].

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Multigraded Shifts of Monomial Ideals

Shamila Bayati
Amirkabir University of Technology, Iran

When we deal with monomial ideals as multigraded modules, the following question arises:

Which properties are shared between the ideal itself and the ideals generated by its multigraded shifts?

More precisely, let $S = k[x_1, \ldots, x_n]$ be the polynomial ring over a field k. We consider this ring with its natural multigrading. Suppose that $I \subseteq S$ is a monomial ideal and consider the ideal $J_k(I) = (\{\mathbf{x}^{\mathbf{a}} \mid \beta_{k,\mathbf{a}}(I) \neq 0\})$ generated by the k-th multigraded shifts of I. It is under question that transferring from I to $J_k(I)$ which properties are preserved.

We first investigate that the property of being (poly)matroidal is inherited by the ideals generated by multigraded shifts.

Regarding this question, we also study Borel ideals and squarefree Borel ideals. A result by Miller and Strumfels shows that the ideal generated by the first multigraded shifts of an equigenerated Borel ideal inherited the property of having linear resolution. We will show this is also the case if we consider the linear quotients property. Moreover, it is shown that if I is a principal Borel ideal or a squarefree Borel ideal, then $J_k(I)$ has linear quotients for each $k = 0, \ldots, \mathrm{pd}(I)$. Furthermore, it is shown that the property of being squarefree Borel is inherited by $J_k(I)$ whenever I is equigenerated.

Some results are based on a joint work with Iman Jahani and Nadiya Taghipour.

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Cartwright-Sturmfels Ideals of Graphs and Linear Spaces

Emanuela De Negri
Università di Genova, Italy

In order to study ideals of minors of multigraded matrices of linear forms, and inspired by a work of Cartwright and Sturmfels, we introduced two classes of ideals in a multigraded polynomial ring, named CS and CS*. It turns out that the ideals of these classes are radical, have nice universal Gröbner bases and good homological properties.

The aim of this talk is to present other classes of CS and CS* ideals, in particular binomial edge ideals, multi-graded homogenizations of linear spaces, and multiview ideals. This approach allows us to recover and generalize recent results of various authors.

These results have been obtained jointly with Aldo Conca and Elisa Gorla.

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A View to Set-theoretically Cohen-Macaulay Ideals

Majid Eghbali
Tafresh University, Iran

In a regular local ring R, an ideal I is *set-theoretically Cohen-Macaulay* if there exists an ideal $J \subset R$ with $\mathrm{rad}(I) = \mathrm{rad}(J)$ such that the ring R/J is Cohen-Macaulay. In this talk, I will report some properties and then give a characterization of this kind of ideals.

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Using Gabriel-Roiter (co)measure for Maximal Cohen-Macaulay Modules

Fahimeh Sadat Fotouhi
University of Isfahan, Iran

The first Brauer-Thrall conjecture asserts that algebras of bounded representation type (meaning that there is a bound on the length of the indecomposable finitely generated modules) have finite type, i.e. the set of isomorphism classes of indecomposable finitely generated modules is finite. This conjecture was solved by Roiter in 1968. The induction scheme which he used in his proof prompted Gabriel to introduce an invariant which Ringel proposed to call Gabriel-Roiter measure. This invariant is defined for any finite length modules. In this talk, we assume that (R, m) is a commutative Cohen-Macaulay local ring. We use of Gabriel-Roiter (co)measure in the category of maximal Cohen-Macaulay R-modules. By using of Gabriel-Roiter (co)measure in this category, we prove the first Brauer-Thrall theorem for the category of maximal Cohen-Macaulay R-modules.

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Results on Linear Resolution and Polymatroidal Ideals

Amir Mafi
Univeirsity of Kurdistan, Iran

Let $R = K[x_1, ..., x_n]$ be the polynomial ring in n variables over a field K and I be a monomial ideal generated in degree d. Bandari and Herzog conjectured (BH-Conjecture) that a monomial ideal I is polymatroidal if and only if all its monomial localizations have linear resolution.

Herzog, Hibi and Zheng proved that if I is a monomial ideal generated in degree 2, then I has a linear resolution if and only if each power of I has a linear resolution. Sturmfels gave an example $I = (def, cef, cdf, cde, bef, bcd, acf, ade)$ with I has a linear resolution while I^2 has no linear resolution. This suggests the following question: Is it true that each power of I has a linear resolution, if I is a squarefree monomial ideal of degree d with I^k has a linear resolution for all $1 \leq k \leq d-1$?

In this talk we speak about BH-Conjecture and we give an affirmative answer in the following cases: (i) height$(I) = n - 1$; (ii) I contains at least $n - 3$ pure powers of the variables $x_1^d, ..., x_{n-3}^d$; (iii) I is a monomial ideal in at most four variables.

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The Gauss Algebra of Toric Algebras

Abbas Nasrollah Nejad
Institute for Advanced Studies in Basic Sciences (IASBS), Iran

Let $S = k[x_0, \ldots, x_d]$ be a polynomial ring over an algebraically closed field k of characteristic zero and $A = k[g_0, \ldots, g_n] \subseteq S$ a k-subalgebra of dimension $d+1$ generated by homogeneous polynomials of the same degree. We define the Gauss algebra $\mathbb{G}(A)$ of A as k-subalgebra of S generated by $d+1$-minors of the Jacobian matrix of $g_0, \ldots, g_n$. The Gauss algebra $\mathbb{G}(A)$ is isomorphic to the coordinate ring of the Gauss image of the projective variety defined parametrically by $g_0, \ldots, g_n$ in the Plücker embedding of the Grassmannian $\mathbb{G}(d, n)$ of d-planes. In this talk, we describe the generators and the structure of $\mathbb{G}(A)$, when A is a Borel fixed algebra or a squarefree Veronese algebra generated in degree 2.

This talk is based on joint work with Jürgen Herzog and Raheleh Jafari.

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Some New Classes of Modules to Tackly Enochs' Conjecture

Zahra Nazemian
IPM, Iran

For an arbitrary ring R and a flat R-module S, we define a class of modules, called the class of S- strongly flat modules, which lies between the class of projective modules and that of flat modules. We will denote this class of modules by $\mathcal{SF}$. For some particular S, we consider some results related to the question whether the fact that the class $\mathcal{SF}$ is a covering class implies that $\mathcal{SF}$ is closed under direct limit. This is a particular case of the so-called "Enochs' Conjecture" (whether covering classes are closed under direct limit). The main problem is determining when this class $\mathcal{SF}$ is a covering class.

This is a joint work with Alberto Facchini.

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Linear Strands of Edge Ideals of Multipartite Uniform Clutters

Amin Nematbakhsh
IPM, Iran

Two techniques are commonly used in the literature to construct linear resolutions of monomial ideals. First is to construct a cellular resolution introduced by Bayer and Sturmfels [2]. The second method is used when the ideal has linear quotients and a regular decomposition function. This technique was introduced by Herzog and Takayama [5]. Based on the work of Yanagawa on squarefree modules one can introduce a third general technique to construct minimal free resolutions of squarefree monomial ideals. This new method was effectively used by D'Alì, Fløystad and the author to construct minimal free resolutions of co-letterplace ideals [3]. Here, we provide yet another example and use this method to construct the first linear strands of edge ideals of d-partite d-uniform clutters, generalizing the construction of resolutions of co-letterplace ideals.

We show that the first linear strand is supported on a relative simplicial complex. Homologies of the dual of linear strands of a squarefree monomial ideal are closely related to the Lyubeznik numbers of its Alexander dual, see [1]. As an application, we show that the Lyubeznik numbers that appear on the last column of the Lyubeznik table of the cover ideal of such clutters are Betti numbers of certain simplicial complexes.

There is a correspondence between edge ideals of d-partite d-uniform clutters and sets of points in the multiprojective space $\mathbb{P}^{\times d}$. A finite set of points in $\mathbb{P}^{\times d}$ is arithmetically Cohen-Macaulay if and only if the corresponding edge ideal has a linear resolution. Recently, Favacchio, Guardo and Migliore gave a characterization for arithmetically Cohen-Macaulay sets of points in the multiprojective space $\mathbb{P}^{\times d}$ [4]. At the end of the talk, we give a translation of their result into a characterization for d-partite d-uniform clutters for which their edge ideals have linear resolutions.

This talk is based on the preprint [6].

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On a Conjecture About Castelnuovo-Mumford Regularity of Binomial Edge Ideals

Mohammad Rouzbahani Malayeri
Amirkabir University of Technology, Iran

Let G be a simple graph on the vertex set $[n]$ and the edge set $E(G)$. Let $S = \mathbb{K}[x_1, \ldots, x_n, y_1, \ldots, y_n]$ be the polynomial ring over a field $\mathbb{K}$. Then the binomial edge ideal of G denoted by J_G is an ideal in S whose generators are all quadrics of the form $f_{ij} = x_i y_j - x_j y_i$, where $\{i, j\} \in E(G)$ and $1 \leq i < j \leq n$. This ideal was introduced by Herzog et al. in [2] and independently by Ohtani in [6].

One of the most interesting numerical invariants in commutative algebra, arising from graded free resolutions, is the Castelnuovo-Mumford regularity. Recall that

$$\operatorname{reg} \frac{S}{J_G} = \max\{j - i : \operatorname{Tor}_i^S(\mathbb{K}, \frac{S}{J_G})_j \neq 0\}.$$

In [5], Matsuda and Murai showed that $\mathcal{L}(G) \leq \operatorname{reg} \frac{S}{J_G} \leq n - 1$, where $\mathcal{L}(G)$ is the sum of the lengths of longest induced paths of connected components of G. Further, they conjectured that $\operatorname{reg} \frac{S}{J_G} \leq n - 2$, if G is not a path. This conjecture was proved in [4]. On the other hand, in [7] the authors showed that $\operatorname{reg} \frac{S}{J_G} \leq c(G)$, for any graph G whose binomial edge ideal admits a quadratic Gröbner basis, where $c(G)$ denotes the number of maximal cliques of G. Afterwards in [8], the following conjecture was formulated:

Conjecture. [8, page 12] Let G be a graph. Then $\operatorname{reg} \frac{S}{J_G} \leq c(G)$.

The aforementioned result by Matsuda and Murai proves this conjecture in the case of trees. In [1], Ene and Zarojanu verified the above conjecture for a class of chordal graphs for which any two maximal cliques intersect in at most one vertex. Later, this result was improved for a bigger subclass of chordal graphs called *generalizad block* graphs [3].

In this talk, among other results, we discuss this conjecture for all chordal graphs.

This talk is based on a joint work with Dariush Kiani and Sara Saeedi Madani.

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Toric Algebras Arising from Cuts in Graphs

Sara Saeedi Madani
Amirkabir University of Technology, Iran

Studying toric algebras and their defining ideals has been and still is an interesting topic of study in commutative algebra and its related areas like algebraic geometry and combinatorics.

In this talk, inspired by conjectures due to B. Sturmfels and S. Sullivant, we look at the so-called cut polytopes which are some convex polytopes attached to graphs. Then, we discuss several algebraic and homological properties of a certain toric algebra associated to such polytopes. We call this algebra the cut algebra of the underlying graph.

This talk is based on a joint work with Tim Römer.

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Lattice Theoretical Approachers for Racks

Amir Saki
Amirkabir University of Technology, Iran

A rack is a set R equipped with a binary operation $\triangleright$ such that for any $a, b, c \in R$, we have $a \triangleright (b \triangleright c) = (a \triangleright b) \triangleright (a \triangleright c)$, and there exists a unique $x \in R$ with $a \triangleright x = b$. Racks have some connections with different branches of mathematics such as knot theory, Hopf algebras, group theory, lattice theory and commutative algebra. This talk is mostly related to the two latter ones. Such an approach was started by I. Heckenberger et al. [1] where they associated a lattice to any rack R called the lattice of subracks of R which consists of all subracks of R. The meet and the join of any two subracks are defined as their intersection and the subrack generated by them, respectively. One can study the lattice of subracks of a rack by considering its order complex and applying topological and commutative algebraic techniques. I. Heckenberger et al. [1] posed some interesting conjectures through their paper. We proved one of them by showing that the lattice of subracks of any rack is atomic [2]. In this talk, we focus on one of the other conjectures. Indeed, we prove that the lattice of subracks of any finite rack is complemented but there are some infinite racks which their lattices of subracks are not complemented.

Let R be a rack. For any $a \in R$, the map which sends each $b \in R$ to $a \triangleright b$ is bijective. The group generated by such maps naturally acts on R. We introduce a new class of racks called G-racks as well. A G-rack is a rack such that any subrack including at least one element from any orbit of the aforementioned action must be the whole R. We also discuss about the homology groups of the order complexes of G-racks. Indeed, we show that for any G-rack R with c orbits the homology groups $\widetilde{H}_i\left(\Delta(R)\right) = 0$ if and only if $i \neq c - 2$.

This talk is based on a joint work with Dariush Kiani.

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Jet Schemes Ideals of Pfaffians

Enrico Sbarra
Università degli Studi di Pisa, Italy

Jet schemes proved to be interesting objects to be studied, as it is shown in several recent results that they carry significant geometric information about the scheme they are associated with. The construction of jet schemes can be easily described in algebraic terms; yet not much is known about their properties and numerical invariants from this point of view. In this talk, after reviewing their definition and some known results, we shall focus on the case of jet schemes ideals associated to determinantal and pfaffians ideals and present some open questions.

This is a joint work with E. De Negri.

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Torsion-free Aluffi Algebras

Zahra Shahidi
Institute for Advanced Studies in Basic Sciences (IASBS), Iran

A special class of algebras which are intermediate between the symmetric and the Rees algebras introduced by P. Aluffi to define characteristic cycle of a hypersurface parallel to the well-known conormal cycle in intersection theory. These algebras are investigated by A. Nasrollah Nejad and A. Simis who named them Aluffi algebras. Let R be a Noetherian ring and $J \subseteq I$ ideals of R. The Aluffi algebra is defined by

$$\mathcal{A}_{R/J}(I/J) := \mathcal{S}_{R/J}(I/J) \otimes_{\mathcal{S}_R(I)} \mathcal{R}_R(I),$$

where $\mathcal{S}_B(\mathfrak{a})$ and $\mathcal{R}_B(\mathfrak{a})$ denote respectively the symmetric and Rees algebra of an ideal $\mathfrak{a}$ in a ring B. There exists a natural surjective R/J-algebra homomorphism $\mathcal{A}_{R/J}(I/J) \twoheadrightarrow \mathcal{R}_{R/J}(I/J)$. A pair of ideals $J \subseteq I \subseteq R$ has been called Aluffi torsion-free if the Aluffi algebra of I/J is isomorphic to the corresponding Rees algebra. In this talk, we give necessary and sufficient conditions for the Aluffi torsion-free property in terms of the first syzygy module of the form ideal J^* in the associated graded ring of I. For two pairs of ideals $J_1, J_2 \subseteq I$ such that $J_1 - J_2 \in I^2$, we prove that if one pair is Aluffi torsion-free the other one is so if and only if the first syzygy modules of J_1 and J_2 have the same form ideals. In the sequel, we introduce the notion of strongly Aluffi torsion-free ideals and present some results on these ideals.

This talk is based on the joint work with A.Nasrollah Nejad and Rashid Zaare-Nahandi.

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Linear Syzygy Graph, Linear Resolution, Linear Quotients and Variable Decomposability

Ali Soleyman Jahan
University of Kurdistan, Iran

Let $S = k[x_1, \ldots, x_n]$ be the polynomial ring in n variables over a field k and I be a monomial ideal in S. We say that I has a d-linear resolution if the graded minimal free resolution of I is of the form:

$$0 \longrightarrow S(-d-p)^{\beta_p} \cdots \longrightarrow S(-d-1)^{\beta_1} \longrightarrow S(-d)^{\beta_0} \longrightarrow I \longrightarrow 0.$$

Let $I \subseteq S$ be a monomial ideal. We denote by $G(I)$ the unique minimal monomial set of generators of I. We say that I has linear quotients if there exists an order $\sigma = u_1, \ldots, u_m$ of $G(I)$ such that the colon ideal $< u_1, \ldots, u_{i-1} >: u_i$ is generated by a subset of the variables, for $i = 2, \ldots, m$. It is known that if I has linear quotients and generated in degree d, then I has a d-linear resolution.

The concept of variable-decomposable monomial ideal was first introduced by Rahmati and Yassemi as a dual concept of vertex-decomposable simplicial complexes. In case that $I = I_{\Delta^\vee}$, they proved that I is variable-decomposable if and only if Δ is vertex-decomposable. Also they proved if a monomial ideal I is variable-decomposable, then it has linear quotients.

In general these three concepts are not equivalent, but for monomial ideals generated in one degree, we have the following implications:
I is variable-decomposable $\Longrightarrow$ I has linear quotients $\Longrightarrow$ I has a linear resolution.

Let G_I be the graphs which its nodes are the generators of I, and two vertices u_i and u_j are adjacent if there exist variables x, y such that $xu_i = yu_j$.

Let $I \subset S$ be a squarefree monomial ideal generated in degree d. We show that if $d = 2$, $d = n - 2$, $n \leq 5$, G_I is cycle and G_I is a tree, then these three concepts are equivalent. As applications of our results, we characterize all Cohen-Macaulay monomial ideals of codimension 2 with a linear resolution.

Let $\Delta = \langle F_1, \ldots, F_m \rangle$ be a simplicial complex. It is shown that Δ is connected in codimension one if and only if $G_{I_{\Delta^\vee}}$ is a connected graph. We show that $I_{\Delta^\vee}$ has linear relations if and only if $\Delta^{(F,G)}$ is connected in codimension one for all facets F and G of Δ. Also, we introduce a simple graph G_Δ on vertex set $\{F_1, \ldots, F_m\}$ which is isomorphic to $G_{I_{\Delta^\vee}}$. As Corollaries of our results, we show that if G_Δ is a cycle or a tree, then the following are equivalent:

(a) Δ is Cohen-Macaulay;

(b) Δ is pure shellable;

(c) Δ is pure vertex-decomposable.

This is a joint work with E. Manouchehri.

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Commutative Algebra and Related Topics, January 16 and 17, 2019
School of Mathematics, IPM, Tehran

Arrangements and the Independence Polynomial

Russ Woodroofe
University of Primorska, FAMNIT, Slovenia

I'll show how to construct a subspace arrangement which encodes the independence polynomial of a graph G, or indeed, the f-vector of an arbitrary simplicial complex. I'll discuss possible applications to unimodality questions.