Abstracts of the Talks

(In Alphabetical Order)

Yuki Arano (University of Kyoto, Japan)

Actions of Tensor Categories

Abstract: I will overview how the quantum group techniques can be applied to the subfactors. First we observe that the classification of subfactors can be interpreted as a classification of actions of tensor categories, which can be seen as a slight generalization of actions of quantum groups and I will present some results on this direction. Then I explain approximation properties of tensor categories which is important in such classification.

References:

[1] Y. Arano: *Unitary spherical representations of Drinfeld doubles*, J. Reine Angew. Math. **742** (2018), 157–186.
[2] Y. Arano: *Comparison of unitary duals of Drinfeld doubles and complex semisimple Lie groups*, Comm. Math. Phys. **351** (2017), no. 3, 1137–1147.
[3] S. K. Ghosh, C. Jones: *Annular representation theory for rigid* C^**-tensor categories*, J. Funct. Anal. **270** (2016), no. 4, 1537–1584.
[4] S. Neshveyev, M. Yamashita: *Drinfeld center and representation theory for monoidal categories*, Comm. Math. Phys. **345** (2016), no. 1, 385–434.
[5] S. Popa, S. Vaes: *Representation theory for subfactors,* λ*-lattices and* C^**-tensor categories*, Comm. Math. Phys. **340** (2015), no. 3, 1239–1280.

Martijn Caspers (TU Delft, the Netherlands)

Applications of Quantum Markov Semi-groups to von Neumann Algebras and Commutator Estimates

Abstract: A quantum Markov semi-group is a continuous time semi-group of normal ucp maps on a von Neumann algebra. Quantum Markov semi-groups are natural analogues of classical Markov processes in quantum probability. In this talk I plan to explain two applications of them: (1) quantum Markov semi-groups provide new tools to study structural results for von Neumann algebras (deformation-rigidity theory), (2) quantum Markov semi-groups can be used as a tool in non-commutative harmonic analysis and we give applications to perturbations of commutators. Though that the results of (1) and (2) may seem quite different at first sight, some of the underlying tools (coming from classical/commutative harmonic analysis) are the same. This will be explained in the talk.

References:

[1] M. Caspers: *Gradient forms and strong solidity of free quantum groups*, arXiv: 1802.01968.
[2] M. Caspers, M. Junge, F. Sukochev, and D. Zanin: *BMO-estimates for non-commutative vector valued Lipschitz functions*.

Uwe Franz (Université de Franche-Comté, France)

The Hochschild Cohomology of Universal Quantum Groups and Related Topics

Abstract: We are interested in the Hochschild cohomology of the Hopf $*$-algebra of a discrete or compact quantum group. A discrete quantum group has the Haagerup property iff there exists a $*$-representation with a proper 1-cocycle, and it has property (T) iff all 1-cocycles of $*$-representations are trivial. The first and second cohomology groups also play a crucial role in the construction and classification of generating functionals and Lévy processes on compact quantum groups. In my talk I will determine residually finite dimensional quotient of the universal unitary quantum group U_Q^+ of Wang and Van Daele. Then I will present new (partial) results about Hochschild cohomology groups of (finite-dimensional) $*$-representations of this quantum group, and discuss some of there implications. This talk is based on joint work with Biswarup Das, Anna Kula, and Adam Skalski.

Paweł Kasprzak (Uniwersytet Warszawski, Poland)

Coideals, Group-like Projections and Idempotent States in Quantum Groups

Abstract: My lecture series is divided into two parts. The first part is based on [1] and [7] (which in turn heavily use [12], [13]) and it is concerned with left coideals subalgebras in a Hopf algebra. Given a Hopf algebra H, its left coideal subalgebra A and a non-zero multiplicative functional μ on A, we can consider the space $L^A_\mu \subset A$ of left μ-integrals, where $L^A_\mu = \{\Lambda \in A : a\Lambda = \mu(a)\Lambda \text{ for all } a \in A\}$. In case of semi-simple A, the elements of L^A_μ can be related with the concept of *shift of a group-like projection* in H. We will show that $\dim L^A_\mu = 1$ if A is a Frobenius algebra, which is the case e.g. for finite dimensional left coideal subalgebras of a weakly finite Hopf algebra. We also prove that if $\dim L^A_\mu > 0$ then $\dim A < \infty$.

Given a group-like element $g \in H$ we define the space $L^g_A \subset A'$ of g-cointegrals on A, where $L^g_A = \{\phi \in A' : (\mathrm{id} \otimes \phi)(\Delta(a)) = g\phi(a) \text{ for all } a \in A\}$. Linking this concept with the theory of μ-integrals we shall prove that

- every semisimple left coideal subalgebra $A \subset H$ which is preserved by the antipode squared admits a faithful 1-cointegral,
- every unimodular finite dimensional left coideal subalgebra $A \subset H$ admitting a faithful 1-cointegral is preserved by the antipode square,
- every non-degenerate right group-like projection in a cosemisimple Hopf algebra is a two sided group-like projection.

The Hopf algebraic part of the lecture will be finished with a list of all ε-integrals in (all) left coideals subalgebras of Taft algebras and the list of all g-cointegrals on them.

The second part of the series is based on [2], [6], [8] and [9] (which in turn heavily use [3], [4], [5], [10], [11]) and it is concerned with the concept of coideals for a locally compact quantum group $\mathbb{G}$. I will first prove the bijective correspondences between the sets of

- idempotent states on $C^u_0(\mathbb{G})$,
- integrable τ-invariant left coideals in $L^\infty(\mathbb{G})$,
- group-like projections in $L^\infty(\hat{\mathbb{G}})$ invariant under the scaling group $\hat{\tau}$.

Using this result I will describe the lattice structure in the set of idempotent states and the duality between the set of normal idempotent states on $\mathbb{G}$ and $\hat{\mathbb{G}}$. If time permits I will explain how the above correspondence extends to a 1-1 correspondence between:

- idempotent contractive functionals on $C_0^u(\mathbb{G})^*$,
- non-degenerate, integrable, $\mathbb{G}$-invariant ternary rings of operators $\mathsf{X} \subset L^\infty(\mathbb{G})$, preserved by the scaling group,
- shifts of group-like projections in $L^\infty(\hat{\mathbb{G}})$ preserved by the scaling group $\hat{\tau}$.

The relations of the above results to the results of Illie-Spronk from classical harmonic analysis will be explained.

References:

[1] A. Chirvasitu, P. Kasprzak, and P. Szulim: *Integrals in coideal subalgebras and group-like projections*, arXiv e-prints, August 2018.

[2] R. Fall, and P. Kasprzak: *Group-like projections for locally compact quantum groups*, Journal of Operator Theory, **80** (2018), 153–166.

[3] U. Franz, and A. Skalski: *A new characterisation of idempotent states on finite and compact quantum groups*, C. R. Math. Acad. Sci. Paris, **347** (2009), 991–996.

[4] U. Franz, and A. Skalski: *On idempotent states on quantum groups*, J. Algebra, **322** (2009), 1774–1802.

[5] U. Franz, A. Skalski, and R. Tomatsu: *Idempotent states on compact quantum groups and their classication on $U_q(2)$, $SU_q(2)$, and $SO_q(3)$*, J. Noncommut. Geom., **7** (2013), 221–254.

[6] P. Kasprzak, and P. M. Sołtan: *The lattice of idempotent states on a locally compact quantum group*, arXive-prints, February 2018.

[7] P. Kasprzak: *Generalized (co)integrals on coideal subalgebras*, ArXive-prints, October 2018.

[8] P. Kasprzak: *Shifts of group-like projections and contractive idempotent functionals for locally compact quantum groups*, International Journal of Mathematics, to be published, 2018.

[9] P. Kasprzak, and F. Khosravi: *Coideals, quantum subgroups and idempotent states*, Quarterly Journal of Mathematics, **68** (2017), 583–615.

[10] M. Neufang, P. Salmi, A. Skalski, and N. Spronk: *Contractive idempotents on locally compact quantum groups*, Indiana Univ. Math. J., **62** (2013), 1983–2002.

[11] P. Salmi, and A. Skalski: *Idempotent states on locally compact quantum groups II*, Quarterly Journal of Mathematics, **68** (2017), 421–431.

[12] S. Skryabin: *Projectivity and freeness over comodule algebras*, Trans. Amer. Math. Soc., **359** (2007), 2597–2623.

[13] S. Skryabin: *Finiteness of the number of coideal subalgebras*, Proc. Amer. Math. Soc., **145** (2017), 2859–2869.

David Kyed (University of Southern Denmark, Denmark)

Uniqueness Questions for C^*-norms on Group Rings

Abstract: I will report on recent joint work with Vadim Alekseev, concerning group rings with a unique Ccompletion. It is easy to see that any locally finite group satisfies this property, and utilizing the socalled Atiyah conjecture we provide some partial evidence for the converse statement.

References:

[1] V. Alekseev, and D. Kyed: *Uniqueness questions for* C^**-norms on group rings*, arXiv e-prints, August 2018.
[2] R. Grigorchuk, M. Musat, and M. Rørdam: *Just-infinite* C^**-algebras*, Comment. Math. Helv. **93** (2018), 157-201.

Issan Patri (Chennai Mathematical Institute, India)

Group Actions on Compact Quantum Groups

Abstract: We will study (discrete) group actions on compact quantum groups and will derive conditions for the action to be ergodic, weak mixing, mixing, compact, etc. We will construct several examples of such actions. We will also see how the crossed product formed by such actions is also a quantum group. If time permits, some questions on the topological structure of automorphism groups of compact quantum groups will also be discussed.

References:

[1] K. Mukherjee, and I. Patri: *Automorphism of compact quantum groups*, Proc. Lond. Math. Soc. **116** (2018), 330–377.
[2] A. Chirvasitu, and I. Patri: *Topological automorphism groups of compact quantum groups*, Math. Z. **290** (2018), 577–598.
[3] S. Wang: *Tensor products and crossed products of compact quantum groups*, Proc. Lond. Math. Soc. **71** (1995), 695–720.

Sven Raum (Stockholm University, Sweden)

C^*-superrigidity

Abstract: A discrete group is called C^*-superrigid if it can be recovered from its reduced group C^*-algebra. In this series of lectures we will motivate this notation by putting it in the context of Higman's unit conjecture and Conne's conjecture on C^*-superrigidity. Having described one basic strategy to show C^*-superrigidity and a recent result with Caleb Eckhardt on C^*-superrigidity of 2-step nilpotent groups, we proceed to explain the latter's proof in detail.

References:

[1] G. Higman: *The units of group-rings*, PhD thesis.
[2] C. Moore, and J. Rosenberg: *Groups with T_1 primitive ideal spaces*, J. Funct. Anal. **22** (1976), 204–224.
[3] S. Echterhoff: *On maximal prime ideals in certain group* C^**-algebras and crossed product algebras*, J. Operator Theory **23** (1990), 317–338.
[4] J. A. Packer, and I. Raeburn: *On the structure of twisted group* C^**-algebras*, Trans. Amer. Math. Soc. **334** (1992), 685–718.
[5] G. A. Elliott: *On the K-theory of the* C^**-algebra generated by projective representation of a torsion-free discrete abelian group*, In Operator algebras and group representations, Vol. I (Neptun, 1980), volume 17 of Monogr. Stud. Math., pages 157184. Pitman, Boston, MA, 1984.
[6] C. Eckhardt, and S. Raum: C^**-superrigidity of 2-step nilpotent groups*, accepted for publication in Adv. Math..

Ebrahim Samei (University of Saskatchewan, Canada)

Exotic C^*-algebras of geometric groups

Abstract: We consider a class of potentially exotic group C*-algebras $C^*(PF_p^*(G))$ for a locally compact group G, and its connection with the class of potentially exotic group C*-algebras $C^*_{L^p}(G)$ introduced by Brown and Guentner [1]. Surprisingly, these two classes of C*-algebras are intimately related. By exploiting this connection, we show $C^*_{L^p}(G) = C^*(PF_p^*(G))$ for $p \in (2,\infty)$, and the C*-algebras $C^*_{L^p}(G)$ are pairwise distinct for $p \in (2,\infty)$ when G belongs to a large class of nonamenable groups possessing the Haagerup property and either the rapid decay property or Kunze-Stein phenomenon by characterizing the positive definite functions that extend to positive linear functionals of $C^*_{L^p}(G)$ and $C^*(PF_p^*(G))$. This greatly generalizes earlier results of Okayasu [4] and Wiersma [6] on the pairwise distinctness of $C^*_{L^p}(G)$ for $2 < p < \infty$ when G is either a noncommutative free group or the group $SL(2,\mathbb{R})$, respectively.

As a byproduct of our techniques, we present two applications to the theory of unitary representations of a locally compact group G. Firstly, we give a short proof of the well-known Cowling-Haagerup-Howe Theorem which presents sufficient condition implying the weak containment of a cyclic unitary representation of G in the left regular representation of G [3]. Also we give a near solution to a 1978 conjecture of Cowling stated in [2]. This conjecture of Cowling states if G is a Kunze-Stein group and π is a unitary representation of G with cyclic vector ξ such that the map

$$G \ni s \mapsto \langle \pi(s)\xi, \xi \rangle$$

belongs to $L^p(G)$ for some $2 < p < \infty$, then $A_\pi \subseteq L^p(G)$. We show $B_\pi \subseteq L^{p+\epsilon}(G)$ for every $\epsilon > 0$ (recall $A_\pi \subseteq B_\pi$). Here A_π and B_π are the closed span of coefficients of π in the norm and w^*-topology of the Fourier-Stieljes algebra $B(G) = C^*(G)^*$, respectively. In particular, we have $A_\pi \subseteq B_\pi$.

This is based on a joint work with M. Wiersma [5].

References:

[1] N. P. Brown and E. P. Guentner, *New C*-completions of discrete groups and related spaces*, Bull. Lond. Math. Soc. 45 (2013), no. 6, 1181-1193.
[2] M. Cowling. The Kunze-Stein phenomenon. Ann. Math. (2), 107:209-234, 1978.
[3] M. Cowling, U. Haagerup and R. Howe, *Almost L^2 matrix coefficients*. J. Reine Angew. Math. 387 (1988), 97-110.

[4] R. Okayasu, *Free group C^*-algebras associated with ℓ_p*. Internat. J. Math. 25 (2014), no. 7, 1450065, 12 pp.

[5] E. Samei and M. Wiersma, *Exotic C*-algebras of geometric groups*, submitted (22 pages), arXiv:1809.07007.

[6] M. Wiersma *L^p-Fourier and Fourier-Stieltjes algebras for locally compact groups.* J. Funct. Anal. 269 (2015), no. 12, 3928-3951.

Roland Vergnioux (Université de Caen Normandie, France)

The Geometry of Free Quantum Groups

Abstract: In the first lecture I will recall the definition of Wang's orthogonal and unitary free quantum groups, as well as Banica's description of their categories of corepresentations. I will also survey the known results about the structure of the associated C and von Neumann algebras. In the second lecture I will discuss the notion of quantum Cayley graph and present some results in the case of free quantum groups. The most interesting object is the edge-reversing operator which plays a central role in the proof (obtained in collaboration with Brannan) that orthogonal free quantum group factors are not isomorphic to free group factors.
In the third lecture I will discuss random walks on discrete quantum group, and more specifically the associated boundaries. In the case of free quantum groups, the Poisson and Martin boundaries can be identified with a quantum analogue of the Gromov boundary which I will also present.

Abstracts of Contributed Talks

Ramin Faal (Ferdowsi University of Mashhad, Iran)

Functions Spaces on Locally Compact Quantum Groups

Abstract: Properties on locally compact group G, can be reflected through function spaces on G like $ap(G)$, $wap(G)$ and $luc(G)$. As a matter of this fact, G is compact if and only if $wap(G) = luc(G)$, or G is amenable if and only if $luc(G)$ is amenable. In this talk we introduce function spaces $ap(L^1(\mathbb{G}))$, $wap(L^1(\mathbb{G}))$ and $luc(L^1(\mathbb{G}))$ on locally compact quantum group $\mathbb{G}$ and characterize them in the language of locally compact quantum groups. Classically, $ap(G)$, $wap(G)$ and $luc(G)$ are m-admissible C*-algebras, thus we attempt to find conditions that force these function spaces to be C*-algebras. In this direction, we characterize the biggest C*-algebra inside $wap(L^1(\mathbb{G}))$ and find a mild condition with which $luc(L^1(\mathbb{G}))$ is a C*-algebra. Next we show taht for a large class of locally compact quantum groups containing Kac algebras $wap(L^1(\mathbb{G}))$ is amenable. Finally, by investigating the relation between function spaces, we characterize compactness and discreteness of locally compact quantum groups. This talk is based on joint work with H. R. E. Vishki.

Fatemeh Khosravi (IPM, Iran)

A Kawada-Itô theorem for locally compact quantum groups

Abstract: We will generalize the concept of compact quantum hypergroup to compact quantum hypersystem. We will consider the condition that guarantee the existance of Haar satates on compact quantum hypersystems. Finally we will prove that all idempotent states on locally compact quantum groups arise as Haar states of compact quantum subhypersystems in a canonical way. This is an appropriate version of Kawada-Itô theorem for locally compact quantum group. This talk is based on an ongoing project with M. Amini.

Mohammad Sadegh Mojahedi Moakhar (Tarbiat Modares University, Iran)

Discrete Quantum Groups

Abstract: We introduce the notion of Zimmer amenability for actions of discrete quantum groups on von Neumann algebras. We prove generalizations of several fundamental results of the theory in the noncommutative case. In particular, we give a characterization of Zimmer amenability of an action $\alpha : \mathbb{G} \curvearrowright N$ in terms of $\hat{\mathbb{G}}$-injectivity of the von Neumann algebra crossed product $N \ltimes_\alpha \mathbb{G}$. As an application we show that the actions of any discrete quantum group on its Poisson boundaries are always amenable.

Abstracts of the Mini Course

(January 5, 2019)

Ramin Faal (Ferdowsi University of Mashhad, Iran)

Compact Quantum Groups

Abstract: We begin by some historical remark which motivates studying quantum groups. We justify why and how quantum groups are defined. Next, we introduce some preliminiaries of coalgebra category. In this stage we can introduce compact quantum groups and describe it's Haar state, comultiplication, multiplicative unitary and representation. Furthermore, some examples of compact quantum groups will be peresented.

References:

[1] U. Franz, A. Skalski, and P. M. Sołtan: *Introduction to compact and discrete quantum groups,* arXiv:1703.10766v1.

[2] A. Maes, and A. Van Daele: *Notes on compact quantum groups,* Nieuw Arch. Wisk. **16** (1998), 73112.

[3] V. Runde: *Characterizations of compact and discrete quantum groups through second duals,* J. Operator Theory **60** (2008), 415-428.

[4] T. Timmermann: *An Invitation to Quantum Groups and Duality,* European Mathematical Society Publishing House, 2008.

Fatemeh Khosravi (IPM, Iran)

Locally Compact Quantum Groups

Abstract: Quantum groups have been studied within several areas of mathematics and mathematical physics. It turns out that there are different approaches to the theory, but we will mainly focus on the approaches taken by J. Kustermans and S. Vaes [1,2,3].
We will start with a short history and motivations behind the theory of locally compact quantum group. Then we will state the definition of locally compact quantum groups in a von Neumann algebraic setting. The multiplicative unitary, which plays a crucial role in the theory, will be introduced. We will explain how the Pontrjagin dual quantum groups is constructed. At the end, the reduced and universal C^*-algebraic approaches will be introduced and the relations between these different approaches will be considered.

References:

[1] J. Kustermans: *Locally compact quantum groups in the universal setting*, Internat. J. Math. **12** (2001), 289–338.
[2] J. Kustermans, and S. Vaes: *Locally compact quantum groups*, Ann. Scient. Éc. Norm. Sup. **33** (2000), 837–934.
[3] J. Kustermans, and S. Vaes: *Locally compact quantum groups in the von Neumann algebraic setting*, Math. Scand. **92** (2003), 68–92.

Mohammad Sadegh Mojahedi Moakhar (Tarbiat Modares University, Iran)

Discrete Quantum Groups

Abstract: This Lecture is devoted to the Pontryagin dual of compact quantum groups, called "discrete quantum groups". We study the comultiplication, counit and Haar measures related to this dual object.

References:

[1] U. Franz, A. Skalski, and P.M. Sołtan: *Introduction to compact and discrete quantum groups*, arXiv:1703.10766.
[2] P. Podleś, and S.L. Woronowicz: *Quantum deformation of Lorentz group*, Comm. Math. Phys. **130** (1991).