Covering Techniques in Representation Theory of Algebras

Razieh Vahed
(University of Khansar)

Email: vahed@ipm.ir

Abstract

Covering technique has been introduced into representation theory of algebras and developed in a series of papers by K. Bongartz, P. Gabriel , C. Riedtmann [BG, G, R] and, E. Green, A. de la Peña, Martinez-Villa, et. al., [Gr, MD]. This technique reduces a problem for modules over an algebra A to that of a category $\mathcal{C}$, often much simpler, with an action of a group G such that A is equivalent to the orbit category $\mathcal{C}/G$.

One of the most important result in this array is Gabriel's theorem [G] which asserts that if $\mathcal{C}$ is a locally bounded $\Bbbk$-category, where $\Bbbk$ is a field, and G is a group with a free action on ind-$\mathcal{C}$, then $\mathcal{C}$ is locally representation-finite if and only if $\mathcal{C}/G$ is so. Here, ind-$\mathcal{C}$ denotes the full subcategory of mod-$\mathcal{C}$ whose objects form a complete set of representatives of isoclasses of indecomposable modules in mod-$\mathcal{C}$ that is closed under the G-action naturally induced from that on $\mathcal{C}$. Then Dowber, Lenzing and Skowroński [DLS] proved a similar relation between $\mathcal{C}$ and $\mathcal{C}/G$ for a wide class of locally bounded categories. In fact, they showed that if $\mathcal{C}$ is a locally support-finite category with a free action of a group G on ind-$\mathcal{C}$, then $\mathcal{C}/G$ is locally support-finite and the Galois covering $P : \mathcal{C} \longrightarrow \mathcal{C}/G$ induces a bijection between the G-orbits of isoclasses of indecomposable objects in mod-$\mathcal{C}$ and the isoclasses of indecomposable objects in mod-$(\mathcal{C}/G)$. Recall that, for a $\mathcal{C}$-module M the support of M, supp$M$, denotes the full subcategory of $\mathcal{C}$ consisting of all objects x of $\mathcal{C}$ such that $M(x) \neq 0$. A locally bounded $\Bbbk$-category $\mathcal{C}$ is called locally support-finite if for every $x \in \mathcal{C}$, the full subcategory of $\mathcal{C}$ formed by the points of all supp$M$, where $M \in$ ind-$\mathcal{C}$ and $M(x) \neq 0$ is finite.

In this series of lectures, we will give an overview of the covering theory for locally bounded categories. Then a Gorenstein version of Gabriel's theorem will be given. We will apply this theorem, to investigate the number of summands in a decomposition of the middle term of almost split sequences of Gorenstein projective modules over monomial algebras.

This talk is based on a joint work with H. Asashiba and R. Hafezi.

References

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