On the index of reducibility in Noetherian modules

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One of the fundamental results in commutative algebra is the irreducible decomposition theorem [1, Satz II and Satz IV] proved by Emmy Noether in 1921. In this paper she had showed that any ideal I of a Noetherian ring R can be expressed as a finite intersection of irreducible ideals, and the number of irreducible ideals in such an irredundant irreducible decomposition is independent of the choice of the decomposition. This number is then called the index of reducibility of I and denoted by $\mathrm{ir}_R(I)$. Although irreducible ideals belong to basic objects of commutative algebra, there are not so much papers on the study of irreducible ideals and the index of reducibility. Then the purpose of this talk is to investigate the index of reducibility of submodules of a finitely generated R-module M as well as the behaviour of the function of indices of reducibility $\mathrm{ir}_M(I^nM)$, where I is an ideal of R, and to present applications of the index of reducibility for the studying the structure of the module M. We show that this function is in fact a polynomial for sufficiently large n. Moreover, we can prove that the big height $\mathrm{bight}_M(I) - 1$ is a lower bound and the analytic spread $\ell_M(I) - 1$ is an upper bound for the degree of this polynomial. However, the degree of this polynomial is still mysterious to us. We can only give examples to show that these bounds are optimal. A classical result of Northcott [2] says that the index of reducibility of a parameter ideal in a Cohen-Macaulay local ring is dependent only on the ring and not on the choice of the parameter ideal. We will generalize Northcott's result and get a characterization for Cohen-Macaulayness of a Noetherian module in terms of the index of reducibility of parameter ideals.

References

[1] E. Noether, Idealtheorie in Ringbereichen, *Math. Ann.* **83** (1921), 24–66.

[2] D.G. Northcott, On irreducible ideals in local rings, *J. London Math. Soc.* **32** (1957), 82–88.