A Brief history of Commutative Algebra in Iran

Commutative Algebra is a popular topic of Mathematics in Iran. Although Algebra has its own roots to a thousand years ago in Iran, in the modern era, Commutative Algebra was founded in Iran in the 1980s by **Hossein Zakeri**. Another influential person in developing this topic was **Siamak Yassemi** whose work had significant effects in extending the involved areas and internationalizing the Iranian community in Commutative Algebra.

The current "Seminar on Commutative Algebra and its related topics" was founded by Yassemi in 2004. The constant support of **IPM**, Institute for Research in Fundamental Sciences in Iran, made this seminar one of the most regular seminars in Iran. The goal of these seminars is to bring together people who are working on Commutative Algebra and related fields, introduce recent developments to the young researchers and Ph.D. students, and to acquaint the young researchers with the new trends in the subject. This seminar has also had the chance to host several great Mathematicians from all over the world. The community of Commutative Algebra in Iran consists of more than 100 Ph.D.s with at least 1500 peer-reviewed publications. During the time, the research interests of Iranian researchers met Homological Methods in Commutative Algebra e.g. Derived Category and Local Cohomology, Combinatorial Commutative Algebra, Representation Theory and Algebraic Geometry.

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Abstracts of the Seminar Talks

(In Alphabetical Order)

The 17th (Virtual) Seminar on
Commutative Algebra and Related Topics,
January 6 - 8, 2021, School of Mathematics, IPM, Tehran

The Socle and the Saturation Number of Powers of c-bounded Stable Ideals

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In recent years there has been a lot of work on algebraic and homological properties of powers of graded ideals in the polynomial ring $S = K[x_1, \dots, x_n]$, where K is a field. Typically, many of the invariants known behave asymptotically well, that is, stabilize or show a regular behavior for sufficiently high powers of I. Let $I \subset S$ be a graded ideal and $\mathfrak{m} = (x_1, \dots, x_n)$ be the unique graded maximal ideal of S . The saturation number of I denoted by $\mathrm{sat}(I)$, is the smallest number ℓ for which $I : \mathfrak{m}^{\ell+1} = I : \mathfrak{m}^{\ell}$. In this talk, we study the socle of $\mathbf{c}$-bounded strongly stable ideals and determine the saturation number of strongly stable ideals and of equigenerated $\mathbf{c}$-bounded strongly stable ideals. We also provide explicit formulas for the saturation number $\mathrm{sat}(I)$ of Veronese type ideals. Using this formula, we show that $\mathrm{sat}(I^k)$ is quasi-linear from the beginning and we determine the quasi-linear function explicitly.

This talk is based on a joint work with Jürgen Herzog and Guangjun Zhu.

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The (IR)Regularity of Tor and Ext

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We investigate the asymptotic behavior of Castelnuovo-Mumford regularity of Ext and Tor, with respect to the homological degree, over complete intersection rings. We derive from a theorem of Gulliksen a linearity result for the regularity of Ext modules in high homological degrees. We show a similar result for Tor, under the additional hypothesis that high enough Tor modules are supported in dimension at most one; we then provide examples showing that the behavior could be pretty hectic when the latter condition is not satisfied.

The talk bases on joint work with Dipankar Ghosh, and Navid Nemati.

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Some Homological Conjectures for Group Rings

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The main aim of this talk is to investigate the well-known Auslander-Reiten Conjecture for group rings. To elaborate a bit more, we recall that a finite dimensional algebra Λ is said to satisfy Auslander-Reiten Conjecture provided any finitely generated Λ-module M satisfying $\operatorname{Ext}^i_\Lambda(M, M \oplus \Lambda) = 0$ for $i \geq 1$ is projective. This was firstly announced by Auslander and Reiten in [2] as a statement that is tightly connected to the generalized Nakayama Conjecture. The conjecture also has commutative algebraic counterparts [1], [4], [5], and has recently been considered in a stronger dual sense [3].

In this talk, we investigate the aforementioned dual conjecture for group rings. We start by looking at group rings raised by finite groups and try to switch to some classes of infinite groups satisfying certain finiteness conditions.

We also try to give some statements on the interplay between the dual version of Auslander-Reiten Conjecture and the dual version of the well-known Moore's conjecture.

The talk is based on a work in progress, joint with Ali Hajizamani.

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Homological Properties of Symbolic Powers of Cover Ideals of Graphs

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Let $S = \mathbb{K}[x_1, \dots, x_n]$ be the polynomial ring in n variables over a field $\mathbb{K}$. To every simple graph G with vertex set $V(G) = \{x_1, \dots, x_n\}$ and edge set $E(G)$, one associates its *edge ideal* $I = I(G)$ defined by

$$I(G) = \big(x_i x_j : \{x_i, x_j\} \in E(G)\big) \subseteq S.$$

The focus of this talk is on the Alexander dual of edge ideals. Namely, the ideal

$$J(G) = \bigcap_{\{x_i,x_j\}\in E(G)} (x_i, x_j),$$

which is called the *cover ideal* of G. The reason for this naming is that $J(G)$ is minimally generated by squarefree monomials corresponding to the minimal vertex covers of G.

We review the recent results about the symbolic powers of cover ideals. In particular, we characterize all graphs G with the property that $J(G)^{(k)}$ has a linear resolution for some (equivalently, for all) integer $k \geq 2$. Also, we determine an upper bound for the regularity of symbolic powers of certain classes of graphs including bipartite graphs, unmixed graphs and claw-free graphs. Furthermore, we compute the largest degree of minimal generators of $J(G)^{(k)}$ when G is either an unmixed of a claw-free graph. Moreover, we study the asymptotic behavior of depth of symbolic powers of cover ideals.

Interested audiences may look at [1, 2] and their references.

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Free Resolutions of Powers of Monomial Ideals

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The question of finding or even effectively bounding the betti numbers of an ideal in a commutative ring is a difficult one. Even more complicated is using the structure of an ideal I to find information about its powers I^r.

Taylor's thesis described a free resolution of any ideal generated by q monomials as the simplicial chain complex of a simplex with q vertices. Taylor's resolution, though often far from minimal, works for *every* monomial ideal I, giving upper bounds for the betti numbers of $\beta_i(I) \leq \binom{q}{i}$ where $\binom{q}{i}$ is the number of i-faces of a q-simplex.

If I is generated by q monomials and r is a positive integer, then I^r can be generated by q^r monomials, and therefore its betti numbers are bounded by $\binom{q^r}{i}$, a number that grows exponentially.

The question that we address in this talk is: can we find a subcomplex of the q^r-simplex whose simplicial chain complex is a free resolution of I^r for any given monomial ideal I generated by q monomials?

We will explore this question by considering "redundant" faces of the Taylor complex of I^r, which will lead us to a (much smaller) subcomplex of the Taylor complex to resolve I^r.

This talk is based on joint work with Susan Cooper, Sabine El Khoury, Sarah Mayes-Tang, Liana M. Şega, and Sandra Spiroff.

Waldschmidt Constants of Genericlines in $\mathbb{P}^3$ and Some Configurations of Points in $\mathbb{P}^N$

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The Waldschmidt constant $\widehat{\alpha}(I)$ of a radical ideal I in the coordinate ring of $\mathbb{P}^N$ measures (asymptotically) the degree of a hypersurface passing through the set defined by I in $\mathbb{P}^N$. Let I be homogeneous ideal in the ring $R = \mathbb{K}[x_0, \cdots, x_N]$. For a positive integer m, let $I^{(m)}$ be the m^{th} symbolic power of I defined as $I^{(m)} = R \cap \bigcap_{P \in Ass(I)} I^m R_P$, where the intersection is taken in the ring of fractions of R and $Ass(I)$ is the set of associated primes of I. Then $\widehat{\alpha}(I) = \lim\limits_{m\to\infty} \frac{\alpha(I^{(m)})}{m}$. Nagata's approach to the 14th Hilbert Problem was based on computing such constant for the set of points in $\mathbb{P}^2$. Since then, these constants drew much attention, but still there are no methods to compute them (except for trivial cases). Therefore the research focuses on looking for accurate bounds for $\widehat{\alpha}(I)$. In this talk we review some know fact about Waldschmidt constant, then in first part we deal with $\widehat{\alpha}(s)$, the Waldschmidt constant for s very general lines in $\mathbb{P}^3$. We prove that $\widehat{\alpha}(s) \geq \lfloor\sqrt{2s-1}\rfloor$ holds for all s, whereas the much stronger bound $\widehat{\alpha}(s) \geq \lfloor\sqrt{2.5s}\rfloor$ holds for all s but $s = 4$, 7 and 10. In the second part we compute Waldschmidt constant for some special configuration of points.
This talk is based on the paper [3].

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Size of Betti Tables of Edge Ideals

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Among the current trends of commutative algebra, the role of combinatorics is distinguished. Especially the combinatorics of finite simple graphs has created fascinating research projects in commutative algebra. Let G be a finite simple graph on the vertex set $[n] = \{1, ..., n\}$ and $E(G)$ the set of edges of G. Recall that a finite simple graph is a finite graph which possesses no loop and no multiple edge together with no isolated vertex. Let $S = K[x_1, ..., x_n]$ denote the polynomial ring in n variables over a field K. The *edge ideal* of G is the ideal $I(G)$ of S which is generated by those monomials $x_i x_j$ with $\{i, j\} \in E(G)$. Since $(S/I(G))$ and $(S/I(G))$ determine the size of the Betti table of the graded minimal free resolution of $S/I(G)$, the question of finding the possible pairs of $(S/I(G))$ and $(S/I(G))$, where G ranges among all finite simple graphs on $[n]$, is attractive and reasonable.

My talk will be a quick survey of [arXiv:2007.14176] with Adam Van Tuyl, *et al.*, and of [arXiv:2002.02523] with Huy Tài Hà. No special knowledge will be required to understand my talk.

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Numerical Semigroup Rings from Relative Point of View

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We investigate algebraic properties of an exponential counterpart of numerical semigroups from a relative point of view. This approach follows Grothendieck's philosophy on algebraic geometry and commutative algebra, which emphasizes homomorphisms rather than rings. The classical study of numerical semigroup rings is a special case of our relative situation. Indeed, for numerical semigroup S there naturally arise two numerical semigroup algebras: (1) The ring $\kappa[\![\mathbf{u}^S]\!]$ is an algebra over a Noether normalization $\kappa[\![\mathbf{u}^s]\!]$, where s is a non-zero element of S. (2) The ring $\kappa[\![\mathbf{u}^S]\!]$ serves also as a coefficient ring for the algebra $\kappa[\![\mathbf{u}]\!]$.

Singularities such as Cohen-Macaulayness, Gorensteiness and complete intersection of the ring $\kappa[\![\mathbf{u}^S]\!]$ are in fact properties of the algebra $\kappa[\![\mathbf{u}^S]\!]/\kappa[\![\mathbf{u}^s]\!]$. One may replace the power series ring $\kappa[\![\mathbf{u}^s]\!]$ by an arbitrary numerical semigroup ring R and consider singularities of a numerical semigroup algebra $\kappa[\![\mathbf{u}^S]\!]/R$. See [2, 3] for investigations emphasizing algebras over a fixed coefficient ring. The purpose of this talk is to clarify notions of the ring $\kappa[\![\mathbf{u}^S]\!]$ that are in fact notions of the algebra $\kappa[\![\mathbf{u}]\!]/\kappa[\![\mathbf{u}^S]\!]$. We replace the power series ring $\kappa[\![\mathbf{u}]\!]$ by an arbitrary numerical semigroup ring and regard it as an algebra over various coefficient rings, [1].

The talk is based on joint work with I-Chiau Huang.

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On the Vanishing of the Normal Hilbert Coefficients of Ideals

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The study of Hilbert-Samuel functions (and polynomials) of filtrations of ideals deals with the numerical characterization of properties of ideals. Several researchers, including M. Nagata, D. G. Northcott, A. Ooishi, S. Itoh, C. Huneke, S. Huckaba, and T. Marley, have studied the interplay between the Hilbert coefficients and properties of associated blowup algebras such as Rees algebras. Over Noetherian local rings, one of the well studied filtration is the normal filtration.

In [5], Itoh gave upper bounds on the first and the second normal Hilbert coefficients, and characterized equality in terms of normal reduction number of the ideal. In the same paper, he proposed the following conjecture:

Conjecture: Let $(R, \mathfrak{m})$ be an analytically unramified Gorenstein local ring of dimension $d \geq 3$. Let I be a parameter ideal. Then $\overline{e}_3(I) = 0$ if and only if $\overline{I^{n+2}} = I^n \overline{I^2}$ for all $n \geq 0$.

We prove a weaker version of this conjecture for higher normal Hilbert coefficients. Using an extension of the techniques from [4] and [5], we explore the relationship between the bounds on the normal Hilbert coefficients and the normal reduction number, via the vanishing of certain graded components of the local cohomology modules.

This is a joint work with Vivek Mukundan and J. K. Verma.

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Gorenstein and Cohen-Macaulay Matching Complexes

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In this talk, K denotes a field and $S = K[x_1, \ldots, x_n]$. Let G be a simple undirected graph on vertex set $V(G) = \{v_1, \ldots, v_n\}$ and edge set $E(G)$. Then the *edge ideal* $I(G)$ of G is the ideal of S generated by $\{x_i x_j | v_i v_j \in E(G)\}$. A graph G is called Cohen-Macaulay (resp. Gorenstein) when $S/I(G)$ is Cohen-Macaulay (resp. Gorenstein) for every field K. Many researchers have tried to combinatorially characterize Cohen-Macaulay (CM, for short) or Gorenstein graphs in specific classes of graphs, see for example, [4, 1, 2] and their references.

Assume that H is a simple undirected graph and $G = \mathrm{L}(H)$ is the *line graph* of H, that is, edges of H are vertices of G and two vertices of G are adjacent if they share a common endpoint in H. Note that the family $\Delta(\mathrm{G})$ of independent sets of G is a simplicial complex whose faces are exactly the matchings of H. Hence $\Delta(\mathrm{G})$ is known and studied as the *matching complex* of H in the literature (see, for example, [3] and the references therein).

Here, we show that the only complete graphs which have a Cohen-Macaulay matching complex are K_m for $m = 1, 2, 3, 5$. Also we show that the matching complex of K_7 is CM over fields with characteristic 0 but is not CM over fields with characteristic 3. Also we investigate combinatorial properties related to Cohen-Macaulayness, such as being connected in codimension 1 or vertex decomposability, in the line graphs of bipartite complete graphs. Moreover, we present a characterization of all graphs with a Gorenstein matching complex and characterize all graphs whose line graphs have a Gorenstein clique complex.

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A Combinatorial Lower Bound for the Depth of Binomial Edge Ideals

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Let G be a graph on $[n]$ and $S = \mathbb{K}[x_1, \dots, x_n, y_1, \dots, y_n]$ be the polynomial ring over a field $\mathbb{K}$. Then, the *binomial edge ideal* associated to G, denoted by J_G, is the ideal in S generated by all the quadratic binomials of the form $f_{ij} = x_i y_j - x_j y_i$, where $\{i, j\} \in E(G)$ and $1 \leq i < j \leq n$. This class of ideals was introduced in 2010 by Herzog, Hibi, Hreinsdóttir, Kahle and Rauh in [2], and independently by Ohtani in [3], as a natural generalization of determinantal ideals, as well as ideals generated by adjacent 2-minors of a $2 \times n$-matrix of indeterminates.

One of the homological invariants associated to binomial edge ideals which is not easy to compute, is *depth*. In [1], Banerjee and Núñez-Betancourt established a nice combinatorial upper bound for the depth of binomial edge ideals in terms of the *vertex connectivity* of the underlying graph. Indeed, for a non-complete connected graph G, they showed that

$$\operatorname{depth} S/J_G \leq n - \kappa(G) + 2, \tag{1}$$

where $\kappa(G)$ denotes the vertex connectivity of G.

In this talk, motivated by the above combinatorial upper bound for the depth of binomial edge ideals, we unveil the first combinatorial lower bound for the depth of S/J_G. Along the way, by a local cohomoligical approach, we employ methods and technics from algebraic topology for characterizing all binomial edge ideals J_G whose $\operatorname{depth} S/J_G \leq 5$.

This talk is based on some joint works with Dariush Kiani and Sara Saeedi Madani.

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The Hessian Polynomial and the Jacobian Ideal of a Reduced Surface in $\mathbb{P}^3$

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For a reduced hypersurface $V(f) \subset \mathbb{P}^n$ of degree d, the Castelnuovo-Mumford regularity of the Milnor algebra $M(f)$ is well understood when $V(f)$ is smooth, as well as when $V(f)$ has isolated singularities. We study the regularity of $M(f)$ when $V(f)$ has a positive dimensional singular locus. In certain situations, we prove that the regularity is bounded by $(d-2)(n+1)$, which is the degree of the Hessian polynomial of f. However, this is not always the case, and we prove that in $\mathbb{P}^3$ the regularity of the Milnor algebra can grow quadratically in d.

The talk bases on joint work with Laurent Busé, Alexandru Dimca, and Gabriel Sticlaru.

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Ulrich Elements in Normal Simplicial Affine Semigroups

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Let $H \subseteq \mathbb{N}^d$ be a normal affine semigroup, $R = K[H]$ its semigroup ring over the field K and ω_R its canonical module. The Ulrich elements for H are those h in H such that for the multiplication map by $\mathbf{x}^h$ from R into ω_R, the cokernel is an Ulrich module. We say that the ring R is almost Gorenstein if Ulrich elements exist in H. For the class of slim semigroups that we introduce, we provide an algebraic criterion for testing the Ulrich propery. When $d = 2$, all normal affine semigroups are slim. Here we have a simpler combinatorial description of the Ulrich property. We improve this result for testing the elements in H which are closest to zero. In particular, we give a simple arithmetic criterion for when is $(1, 1)$ an Ulrich element in H.

This is joint work with Jürgen Herzog and Raheleh Jafari, see arxiv:1909.06846.

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Primary Decompositions and Powers of Ideals

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This talk is about associated primes of powers of an ideal in Noetherian commutative rings. Brodmann proved that the set of associated primes stabilizes for large powers. In general, the number of associated primes can go up or down as the exponent increases. This talk is about ways of computing associated primes and about sequences $\{a_n\}$ for which there exists an ideal I in a Noetherian commutative ring R such that the number of associated primes of R/I^n is a_n.

This is a report on four separate projects with Sarah Weinstein, Jesse Kim, Robert Walker, and ongoing work with Roswitha Rissner.

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Investigating the Small Cohen-Macaulay Conjecture

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Let $(R, \mathfrak{m}_R)$ be a local ring. An R-module (algebra) B is said to be a balanced big Cohen-Macaulay module (algebra) provided every system of parameters of R is a regular sequence on B and $B/\mathfrak{m}_R B \neq 0$. When such a module B is a finitely generated module we say that B is a maximal Cohen-Macaulay module. Although the celebrated conjecture on the existence of balanced big Cohen-Macaulay modules (algebras) has been settled affirmatively since 2016, the conjecture that every complete local ring admits a maximal Cohen-Macaulay module, so-called the Small Cohen-Macaulay Conjecture, is still widely open even in dimension 3 and even for rings containing a field.

In this talk, we report the results of our investigation on the Small Cohen-Macaulay Conjecture in a joint work with Kazuma Shimomoto. For example, we present (and we discuss) the following reduction of the conjecture:

Let $(A, \mathfrak{m}_A)$ be a complete regular ring and $X_1, \ldots, X_n$ indeterminates over A and consider the excellent regular local ring $A' := A[X_1, \ldots, X_n]_{(\mathfrak{m}_A, X_1, \ldots, X_n)}$. Let $\mathfrak{p}$ be a prime almost complete intersection ideal of A'. If the factorial extended Rees algebra of A' w.r.t. $\mathfrak{p}$ (or its ordinary Rees algebra) admits a graded maximal Cohen-Macaulay module then the Small Cohen-Macaulay Conjecture holds. In other words, roughly speaking, one can say that the Small-Cohen-Macaulay Conjecture reduces to the graded modules over (factorial) blow-up algebras of excellent regular local rings.

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The 17th (Virtual) Seminar on
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Numerical Criteria for Integral Dependence: The Multiplicity Sequence

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The talk deals with multiplicity based criteria for the integral dependence of two arbitrary ideals $I \subset J$ of an equidimensional and universally catenary Noetherian local ring. We focus on a criterion that uses the multiplicity sequence introduced by Achilles and Manaresi.

For the basics, Sections 1.1-1.3, 8.1-8.3 and 11.1-11.3 of the book by Swanson-Huneke, *Integral Closure of Ideals, Rings, and Modules* could be useful.

This is a report on joint work with Claudia Polini, Ngo Viet Trung, and Javid Validashti [1].

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A Relative Silting Theorem

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Tilting theory is a topic in representation theory of algebras and its fundamental idea is to relate the module categories to their derived categories of two algebras. One of the important results in this theory, the tilting theorem, was proved by Brenner and Butler [BB] that states a tilting module T over an algebra Λ induces two torsion pairs, one in the category of Λ-modules and the other one in the category of $\mathrm{End}_\Lambda(T)$-modules in conjunction with a pair of crosswise equivalences between the torsion and torsion-free classes.

The notion of tilting modules was introduced by Happel and Ringel in [HR]. Then Rickard [R] introduced the concept of tilting complexes and developed Morita theory for derived categories of module categories. The concept of tilting complexes has been generalized in different directions. One of them is due to Keller and Vossieck [KV] and involves the notion of silting complexes.

In [BZ], Buan and Zhou generalized the tilting theorem to a silting theorem for a 2-term silting complexes over a finite dimensional algebra Λ. In this talk, we give a relative version of a silting theorem for any abelian category which is a finite R-variety for some commutative Artinian ring R.

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