ABSTRACTS AND REFERENCES

LARS WINTHER CHRISTENSEN

1. Differential Graded Algebra Structures on Free resolutions

Free resolutions are fundamental in homological studies of modules, and whenever a free resolution can be endowed with additional structures of, say, algebraic or combinatorial nature, new tools can be brought to bear on their study. In this talk I will survey the possibilities and limitations to endowing free resolutions in local algebra with differential graded algebra structures.

Primary references [1, 2, 5, 6, 15, 16, 17, 19]

2. Free Resolutions of Length 3

Every free resolution of length 3 over a local ring has a differential graded algebra structure. This structure on the resolution induces a graded-commutative algebra structure in homology, which can be used to classify cyclic modules of projective dimension 3 and, via Cohen's structure theorem, local rings of codepth 3. This classification project was started in the late 1980s. I will explain it and discuss the current status of affairs.

Primary references [3, 4, 6, 7, 9, 11, 10, 12, 14, 21]

3. Generic Artinian Quotients of the Trivariate Polynomial Algebra

There is empirical evidence that artinian quotients of the polynomial ring in 3 variables over a field fall on a spectrum between Gorenstein and Golod. I will show how differential graded algebra structures on free resolutions play a key role in explaining this observed behavior.

Primary references [3, 8, 11, 13, 18, 20]

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References

[1] Luchezar L. Avramov, *Obstructions to the existence of multiplicative structures on minimal free resolutions*, Amer. J. Math. **103** (1981), no. 1, 1–31. MR0601460

[2] Luchezar L. Avramov, *Infinite free resolutions*, Six lectures on commutative algebra (Bellaterra, 1996), Progr. Math., vol. 166, Birkhäuser, Basel, 1998, pp. 1–118. MR1648664

[3] Luchezar L. Avramov, *A cohomological study of local rings of embedding codepth 3*, J. Pure Appl. Algebra **216** (2012), no. 11, 2489–2506. MR2927181

[4] Luchezar L. Avramov, Andrew R. Kustin, and Matthew Miller, *Poincaré series of modules over local rings of small embedding codepth or small linking number*, J. Algebra **118** (1988), no. 1, 162–204. MR0961334

[5] Winfried Bruns and Jürgen Herzog, *Cohen-Macaulay rings*, Cambridge Studies in Advanced Mathematics, vol. 39, Cambridge University Press, Cambridge, 1993. MR1251956

[6] David A. Buchsbaum and David Eisenbud, *Algebra structures for finite free resolutions, and some structure theorems for ideals of codimension* 3, Amer. J. Math. **99** (1977), no. 3, 447–485. MR0453723

[7] Lars Winther Christensen, Luigi Ferraro, Francesca Gandini, Frank Moore, and Oana Veliche, *ResLengthThree: Multiplication in free resolutions of length three. Version 1.0*, A *Macaulay2* package available at `https://github.com/Macaulay2/M2/tree/master/M2/Macaulay2/packages`.

[8] Lars Winther Christensen and Oana Veliche, *Generic local rings on a spectrum between Golod and Gorenstein*, Preprint arXiv:2105.13167 [math.AC].

[9] Lars Winther Christensen and Oana Veliche, *TorAlgebra: classification of local rings based on multiplication in homology. Version 2.1*, A *Macaulay2* package available at `https://github.com/Macaulay2/M2/tree/master/M2/Macaulay2/packages`.

[10] Lars Winther Christensen and Oana Veliche, *Local rings of embedding codepth 3: a classification algorithm*, J. Softw. Algebra Geom. **6** (2014), 1–8. MR3338667

[11] Lars Winther Christensen and Oana Veliche, *Local rings of embedding codepth 3. Examples*, Algebr. Represent. Theory **17** (2014), no. 1, 121–135. MR3160716

[12] Lars Winther Christensen, Oana Veliche, and Jerzy Weyman, *Three takes of almost complete intersection ideals of grade* 3, to appear, Commutative Algebra, Springer, 2022. Preprint arXiv:2106.14674 [math.AC].

[13] Lars Winther Christensen, Oana Veliche, and Jerzy Weyman, *Trimming a Gorenstein ideal*, J. Commut. Algebra **11** (2019), no. 3, 325–339. MR4038053

[14] Lars Winther Christensen, Oana Veliche, and Jerzy Weyman, *Linkage classes of grade 3 perfect ideals*, J. Pure Appl. Algebra **224** (2020), no. 6, 106185, 29 pp. MR4048506

[15] Tor Holtedahl Gulliksen, *A proof of the existence of minimal R-algebra resolutions*, Acta Math. **120** (1968), 53–58. MR0224607

[16] Andrew R. Kustin, *Gorenstein algebras of codimension four and characteristic two*, Comm. Algebra **15** (1987), no. 11, 2417–2429. MR0912779

[17] Andrew R. Kustin, *The minimal resolution of a codimension four almost complete intersection is a DG-algebra*, J. Algebra **168** (1994), no. 2, 371–399. MR1292771

[18] Andrew R. Kustin, Liana M. Şega, and Adela Vraciu, *Poincaré series of compressed local Artinian rings with odd top socle degree*, J. Algebra **505** (2018), 383–419. MR3789918

[19] Andrew R. Kustin and Matthew Miller, *Algebra structures on minimal resolutions of Gorenstein rings of embedding codimension four*, Math. Z. **173** (1980), no. 2, 171–184. MR0583384

[20] Keller Vandebogert, *Structure theory for a class of grade* 3 *homogeneous ideals defining type* 2 *compressed rings*, J. Commut. Algebra, to appear. Preprint arXiv:1912.06949 [math.AC]; 24 pp.

[21] Jerzy Weyman, *On the structure of free resolutions of length* 3, J. Algebra **126** (1989), no. 1, 1–33. MR1023284

L.W.C. Texas Tech University, Lubbock, TX 79409, U.S.A.
Email address: `lars.w.christensen@ttu.edu`
URL: `http://www.math.ttu.edu/~lchriste`