Some New Variations of Auslander's Formula and Applications

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Let $\mathcal{C}$ be an abelian category. A contravariant functor F from $\mathcal{C}$ to the category of abelian groups $\mathcal{A}b$ is called finitely presented, or coherent [A], if there exists an exact sequence

$$\mathrm{Hom}_{\mathcal{C}}(-, X) \longrightarrow \mathrm{Hom}_{\mathcal{C}}(-, Y) \longrightarrow F \longrightarrow 0$$

of functors. Let $\mathrm{mod}\mathcal{C}$ denote the category of all coherent functors. The systematic study of $\mathrm{mod}\mathcal{C}$ is initiated by Auslander [A]. He, not only showed that $\mathrm{mod}\mathcal{C}$ is an abelian category of global dimension less than or equal to two but also provided a nice connection between $\mathrm{mod}\mathcal{C}$ and $\mathcal{C}$. This connection, which is known as Auslander's formula [L, K], suggests that one way of studying $\mathcal{C}$ is to study $\mathrm{mod}\mathcal{C}$, that has nicer homological properties than $\mathcal{C}$, and then translate the results back to $\mathcal{C}$. In particular if we let $\mathcal{C}$ to be $\mathrm{mod}\Lambda$, where Λ is an artin algebra, Auslander's formula translates to the equivalence

$$\frac{\mathrm{mod}(\mathrm{mod}\Lambda)}{\{F \mid F(\Lambda) = 0\}} \simeq \mathrm{mod}\Lambda$$

of abelian categories. As it is mentioned in [L], 'a considerable part of Auslander's work on the representation theory of finite dimensional, or more general artin, algebras can be connected to this formula'.

Recently, Krause [K] established a derived version of this equivalence. In my talk, some different (relative and derived) versions of this formula will be explained. Then I will give some applications of our results for artin algebras.

References

[A] M. Auslander, *Coherent functors,* 1966 Proc. Conf. Categorical Algebra (La Jolla, Calif., 1965) pp. 189-231 Springer, New York.

[L] H. Lenzing, *Auslander's work on Artin algebras,* Algebras and modules, I (Trondheim, 1996), 83-105, CMS Conf. Proc., 23, Amer. Math. Soc., Providence, RI, 1998.

[K] H. Krause, *Deriving Auslander's formula,* Doc. Math. **20** (2015) 669-688.