Cohen-Montgomery duality for bimodules and its application to stable equivalences of Morita type

Hideto Asashiba
(Faculty of Science, Shizuoka University)

Abstract

We fix a commutative ring $\Bbbk$ and a group G. To include infinite coverings of $\Bbbk$-algebras into consideration we usually regard $\Bbbk$-algebras as locally bounded $\Bbbk$-categories with finite objects, so we will work with small $\Bbbk$-categories. For small $\Bbbk$-categories R and S with G-actions we introduce *G-invariant S-R-bimodules* and their category denoted by $S\text{-Mod}^G\text{-}R$, and denote by R/G the orbit category of R by G, which is a small G-graded $\Bbbk$-category. For small G-graded $\Bbbk$-categories A and B we introduce *G-graded B-A-bimodules* and their category denoted by $B\text{-Mod}_G\text{-}A$, and denote by $A\#G$ the smash product of A and G, which is a small $\Bbbk$-category with G-action. Then the Cohen-Montgomery duality theorem [2, 1] says that we have equivalences $(R/G)\#G \simeq R$ and $(A\#G)/G \simeq A$, by which we identify these pairs. In the talk we introduce functors $(\text{-})/G : S\text{-Mod}^G\text{-}R \to (S/G)\text{-Mod}_G\text{-}(R/G)$ and $(\text{-})\#G : A\text{-Mod}_G\text{-}B \to (A\#G)\text{-Mod}^G\text{-}(B\#G)$, and show that they are equivalences and quasi-inverses to each other (by applying $A := R/G$, $R := A\#G$, etc.), have good properties with tensor products and preserve projectivity of bimodules. We apply this to equivalences given by bimodules such as Morita equivalences and stable equivalences of Morita type to have the following theorem:

Theorem. (1) *There exists a "G-invariant stable equivalence of Morita type" between R and S if and only if there exists a "G-graded stable equivalence of Morita type" between R/G and S/G.*

(2) *There exists a "G-graded stable equivalence of Morita type" between A and B if and only if there exists a "G-invariant stable equivalence of Morita type" between $A\#G$ and $B\#G$.*

Here we note that a G-invariant (resp. G-graded) stable equivalence of Morita type is defined to be a usual stable equivalence of Morita type with additional properties, and does not mean an equivalence between stable categories of G-invariant (resp. G-graded) modules. The corresponding result for standard derived equivalences holds under the condition that idempotents in R/G, S/G (resp. A, B) split for "if part" in (1) (resp. "only if part" in (2)).

References

[1] Asashiba, H.: *A generalization of Gabriel's Galois covering functors II: 2-categorical Cohen-Montgomery duality*, preprint arXiv: 0905.3884.

[2] Cohen, M. and Montgomery, S.: *Group-graded rings, smash products, and group actions*, Trans. Amer. Math. Soc. **282** (1984), 237–258.