اصول غیر کلاسیک در ریاضیات ساختی

مجتبی آقایی
دانشگاه صنعتی اصفهان

در این سخنرانی، اصولی که با اضافه شدن به منطق شهودی زیربنای رویکردهای مختلف ریاضیات ساختی را تشکیل می دهند در مدلهای کریپکی و بث بررسی می شوند. این اصول از دید کلاسیک نادرستند یا دارای توجیه ساختی غیر کلاسیک هستند. کاربرد این اصول در ساخت آنالیز ساختی و توجیهات فلسفی آن نیز بررسی خواهد شد.

Some Aspects of Continues Logic

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We compare aspects of definability and type space in full and linear continuous logics. We then prove some definability theorems such as Beth, Svenonus and Herbrand in the linear one.

منطق های زیرساختی و ترجمه های منفی

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در این سخنرانی با معرفی زبان منطق محمولی پایه و سیستم حساب رشته ها برای آن، به بررسی ترجمه های منفی روی توسیع های این منطق خواهیم پرداخت. ترجمه های منفی کولموگروف، گودل-گنزن، کورودا و توسیعی از ترجمه کورودا را معرفی می کنیم و نشان می دهیم ترجمه های منفی کولموگروف و گودل-گنزن روی منطق محمولی پایه با یکدیگر معادلند.

مراجع.

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Labeled Sequent Calculus: The case of Modal Logic S5

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This paper introduces a cut-free labeled sequent calculus for S5. We internalize the universal models (Kripke models without accessibility relation) of S5 within the syntax of sequent calculus to produce a labeled sequent calculus for S5. We show that the labeled system enjoys subformula property, all rules are invertible and the structural rules (weakening, contraction, and cut) are admissible. Finally soundness, completeness and termination of proof search are established.

Keywords: Modal logic, Labeled sequent calculus, Cut elimination.

Trees in set theory: Aronszajn, Souslin, Kurepa and almost Souslin

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The theory of trees forms a significant and highly interesting part of set theory, related to various other areas of mathematics, both within set theory proper and without. We give a survey of some known results about trees, especially about Aronszajn, Souslin, Kurepa and almost Souslin trees. We will discuss their existence in L, the Godel's constructible universe, and in forcing extensions. Finally we will present some of recent works and state some known open problems.

Logic in a Topos

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We first introduce the notion of a topos. Then we show that, in a topos, the set of subobjects of a given object is a Heyting algebra and that the power object of a given object is an internal Heyting algebra. Finally we introduce the Mitchell-Benabou language and give several illustrating examples.

Keywords: topos, Heyting algebra, Mitchell-Benabou language.

Canonical Ramsey Theorem with a largeness condition

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For a function f with domain $[X]^n$, where $X \subseteq \mathbb{N}$, we say that $H \subseteq X$ is *canonical* for f if there is a $\upsilon \subseteq n$ such that for any $x_0, \ldots, x_{n-1}$ and $y_0, \ldots, y_{n-1}$ in H, $f(x_0, \ldots, x_{n-1}) = f(y_0, \ldots, y_{n-1})$ iff $x_i = y_i$ for all $i \in \upsilon$. Canonical Ramsey Theorem is the statement that for any $n \in \mathbb{N}$, if $f : [\mathbb{N}]^n \to \mathbb{N}$, then there is an infinite $H \subseteq \mathbb{N}$ canonical for f. In this talk we study the finite version of the Canonical Ramsey Theorem with a largeness condition.

اندازه هار و منطق انتگرال

کریم خانکی
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با استفاده از ایده های اثبات قضیه وجودی کولوموگروف در منطق انتگرال، به مطالعه و اثبات قضیه های وجودی در نظریه اندازه، احتمالات و ارگودیک می پردازیم. بدین منظور، با استفاده از قضیه فشردگی، نشان می دهیم که وجود یک اندازه با ویژگی های مطلوب روی یک فضای هاسدورف فشردهX ، معادل با وجود اندازه هایی با ویژگی های مطلوب روی Xبرای هر تعداد متناهی از آن ویژگی ها می باشد. با این روش شرط های معادلی برای وجود اندازه های پایا روی فضاهای فشرده می یابیم که ما را به سوی ایده ها و روش های جدیدی برای اثبات قضیه های وجودی هدایت می کند و از این شیوه، وجود و ویژگی های اندازه های هار را مورد بررسی قرار می دهیم. این ایده ها ما را به سوی اثبات جدیدی از یک قضیه اساسی در نظریه ارگودیک و همچنین اثبات ها و تعمیم های جدیدی از قضایای نقطه ثابت مانند قضیه نقطه ثابت مارکوف-کاکوتانی و دستاوردهای دیگری در آنالیز تابعی سوق می دهد.

مقدمه‌ای بر مجموعه‌های فازی

ماشاا... ماشین‌چی

دانشگاه شهید باهنر کرمان

در این سخنرانی در مورد مطالب زیر بحث خواهیم کرد. تاریخچه‌ای مختصر از مجموعه‌های فازی، مفهوم مجموعه‌های فازی و اعمال جبری روی آنها، اصل تجزیه، اصل گسترش و معرفی کاربردی از مجموعه‌های فازی.

Some of recent trends in stability and NIP theories

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Abstract

In his program for the classification of first order theories, S. Shelah introduced and developed a machinery called stability theory. Also he introduced several dividing lines in the class of first order theories on the base of the particular combinatorial complexities of the theories. Stable theories, NIP theories and simple theories are among the most important of these classes. Later on, several people extended the theory and applied it to the different parts of mathematics. Currently this topic is an active part of the model theory and developing techniques in some of the mentioned classes such as NIP class is highly under consideration. In this lecture we talk about some of the main notions, theorems and techniques on which the machinery of stability and its newer versions (sometimes called the neo-stability theory) are based.

Decidability and Undefinability: A Case for Quantifier Elimination

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Providing a recursively enumerable set of axioms for a mathematical structure is equivalent to giving an algorithm for deciding the theory of that structure: given a first-order formula φ of the language $\mathcal{L}$, either φ or $\neg\varphi$ is true in the structure $\mathscr{M} = (M, \mathcal{L})$, so exactly one of them belongs to the consequences of the axioms, which is a recursively enumerable set. Thus, the theory of $\mathscr{M}$ is r.e. and co-r.e., hence decidable; or in the other words, one can decide for a give formula φ whether $\mathscr{M} \models \varphi$ or not.

A well-known theorem of Cantor states that any two countable dense linear orders without endpoints are isomorphic. Thus $(\mathbb{Q}, <)$ is the only countable model for the theory of "dense linear orders without endpoints", and so this theory is complete. Whence, the first-order theories of $(\mathbb{Q}, <)$ and $(\mathbb{R}, <)$ are identical, though we know that these two structures are completely different (the latter is complete in the sense that any subset which is bounded above has a supremum, while the former is not).

Putting another way, one can say that the theory of "dense linear orders without endpoints" completely axiomatizes the first-order theories of the structures $(\mathbb{Q}, <)$ and $(\mathbb{R}, <)$, and so these structures are decidable. Another way of proving this fact is by "Quantifier Elimination" which is a helpful tool for establishing decidability of mathematical structures, and also for showing the undefinability of certain sets in structures. One can also show that the theory of "discrete orders without endpoints" completely axiomatizes the theory of $(\mathbb{Z}, <)$, and the theory of $(\mathbb{N}, <)$ can be axiomatized as "discrete order with a least point and no last point".

This settles the theory of order in the sets $\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$. In this talk we will study the theories of the following structures:

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$	$\mathbb{C}$
$\{<\}$	$\langle\mathbb{N}, <\rangle$	$\langle\mathbb{Z}, <\rangle$	$\langle\mathbb{Q}, <\rangle$	$\langle\mathbb{R}, <\rangle$	–
$\{+\}$	$\langle\mathbb{N}, +\rangle$	$\langle\mathbb{Z}, +\rangle$	$\langle\mathbb{Q}, +\rangle$	$\langle\mathbb{R}, +\rangle$	$\langle\mathbb{C}, +\rangle$
$\{\cdot\}$	$\langle\mathbb{N}, \cdot\rangle$	$\langle\mathbb{Z}, \cdot\rangle$	$\langle\mathbb{Q}, \cdot\rangle$	$\langle\mathbb{R}, \cdot\rangle$	$\langle\mathbb{C}, \cdot\rangle$
$\{+, <\}$	$\langle\mathbb{N}, +, <\rangle$	$\langle\mathbb{Z}, +, <\rangle$	$\langle\mathbb{Q}, +, <\rangle$	$\langle\mathbb{R}, +, <\rangle$	–
$\{+, \cdot\}$	$\langle\mathbb{N}, +, \cdot\rangle$	$\langle\mathbb{Z}, +, \cdot\rangle$	$\langle\mathbb{Q}, +, \cdot\rangle$	$\langle\mathbb{R}, +, \cdot\rangle$	$\langle\mathbb{C}, +, \cdot\rangle$
$\{\cdot, <\}$	$\langle\mathbb{N}, \cdot, <\rangle$	$\langle\mathbb{Z}, \cdot, <\rangle$	$\langle\mathbb{Q}, \cdot, <\rangle$	$\langle\mathbb{R}, \cdot, <\rangle$	–
$\{+, \cdot, <\}$	$\langle\mathbb{N}, +, \cdot, <\rangle$	$\langle\mathbb{Z}, +, \cdot, <\rangle$	$\langle\mathbb{Q}, +, \cdot, <\rangle$	$\langle\mathbb{R}, +, \cdot, <\rangle$	–

Surprisingly, we will see that some theories in this table are missing in the literature; i.e., have not been studied before. One example is the theory $(\mathbb{Q}, \cdot)$ which is decidable, but no proof of it can be found. The theories $(\mathbb{R}, \cdot)$ and $(\mathbb{C}, \cdot)$ are also decidable, because by a theorem of Tarski, the structures $(\mathbb{R}, +, \cdot)$ and $(\mathbb{C}, +, \cdot)$ are decidable. We give a new proof for the decidability of $(\mathbb{R}, \cdot)$ and $(\mathbb{C}, \cdot)$ without appealing to Tarski's result. Also a new proof for the decidability of the structure $(\mathbb{R}, \cdot, <)$ can be given without using Tarski's theorem. We also show that the theory $(\mathbb{Z}, \cdot, <)$ is undecidable, and the theory $(\mathbb{Q}, \cdot, <)$ is decidable. These are new theorems with novel and nontrivial proofs. All in all we complete the picture as below, where decidability is indicated by Δ_1 and undecidability by $\not\Delta_1$:

	$\mathbb{N}$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R}$	$\mathbb{C}$
$\{<\}$	Δ_1	Δ_1	Δ_1	Δ_1	–
$\{+\}$	Δ_1	Δ_1	Δ_1	Δ_1	Δ_1
$\{\cdot\}$	Δ_1	Δ_1	Δ_1	Δ_1	Δ_1
$\{+, <\}$	Δ_1	Δ_1	Δ_1	Δ_1	–
$\{+, \cdot\}$	$\not\Delta_1$	$\not\Delta_1$	$\not\Delta_1$	Δ_1	Δ_1
$\{\cdot, <\}$	$\not\Delta_1$	$\not\Delta_1$	Δ_1	Δ_1	–

At the end, we will discuss some new results and some open problems when the exponential function is added into the language.

Computable Analysis and Some Applications

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Abstract

Computable analysis is a branch of computability theory studying those functions on the real numbers and related sets which can be computed by machines such as digital computers. The increasing demand for reliable software in scientific computation and engineering requires a sound and broad foundation not only of the analytical/numerical but also of the computational aspects of real number computation. The central subject of this approach of computable analysis is "Type-2 Theory of Effectivity" (TTE), one of the approaches to effective analysis being discussed today. It is based on definitions of computable real numbers and functions by A. Turing, A. Grzegorczyk and D. Lacombe. A framework of concrete computability on finite and infinite sequences of symbols is introduced. Computability on finite and infinite sequences of symbols can be transferred to other sets by using them as names. First, computability induced by naming systems is discussed. Then, computable real numbers and functions are introduced. Also, computability on some special spaces as metric spaces is discussed. Afterward, Some applications of this method to study effectivity of metric model theory and measure theory are presented.