One Day Conference on Algebraic Graph Theory

Saturday, October 27, 2018

10:00 – 11:00 ***Integral Cayley graphs***
Elena Konstantinova
Sobolev Institute of Mathematics, Novosibirsk State University

11:00- 12:00 ***Strictly Deza graphs and their new constructions***
Vladislav Kabanov
Institute of Mathematics and Mechanics UB RAS

12:00 **Coffee Break**

14:00 -15:00 ***Dual Seidel switching and Deza graphs***
Leonid Shalaginov
Chelyabinsk State University & N.N. Krasovskii Institute of Mathematics and Mechanics (IMM UB RAS)

15:00-16-00 ***On vertex connectivity of Deza graphs***
Sergey Goryainov
Shanghai Jiao Tong University & Krasovskii Institute of Mathematics and Mechanics

16:00 **Coffee Break**

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On vertex connectivity of Deza graphs

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We consider finite undirected graphs without loops and multiple edges.

A graph is called *k-regular*, if each of its vertices has exactly k neighbours. A k-regular graph with n vertices is called a *Deza* graph with parameters (n, k, b, a), $b \geq a$, if the number of common neighbours of any two of its vertices takes precisely two values a or b. A Deza graph is called a *strictly Deza* graph, if it has diameter 2 and is not strongly regular. *The vertex connectivity* $\kappa(\Gamma)$ of a connected graph Γ is the minimum number of vertices one has to remove in order to make the graph Γ disconnected (or empty).

In 1985, Brouwer and Mesner proved that the vertex connectivity of a strongly regular graph is equal to its valency. In 2009, Brouwer and Koolen generalized this result to the class of distance-regular graphs.

In this talk we discuss several results on the vertex connectivity of strictly Deza graphs and related open problems.

Strictly Deza Graphs and Their New Constructions

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This is joint work with Sergey Goryainov,
Willem H. Haemers, and Leonid Shalaginov

A k-regular graph Γ on n vertices is called a *Deza graph* with parameters (n, k, b, a) if the number of common neighbours of two distinct vertices takes on only two values a or b ($a \leq b$). The concept of Deza graphs was introduced in the initial paper [1].

If the number of common neighbours of two vertices only depend on whether the vertices are adjacent or not, then Γ is a strongly regular graph with parameters (n, k, λ, μ), where λ (μ) is the number of common neighbours of two adjacent (non-adjacent) vertices; so $\{a, b\} = \{\lambda, \mu\}$. A Deza graph is called a *strictly Deza graph* if it has diameter 2 and is not strongly regular. In [1] a basic theory of strictly Deza graphs was developed and several ways to construct such graphs were introduced. Moreover, all strictly Deza graphs with number of vertices at most 13 were found.

Some problems arising in the theory of strictly Deza graphs are similar to those in the theory of strongly regular graphs. However, results and methods in these theories sometimes differ, and an analysis of these differences can enrich both theories. We discuss the latest results of strictly Deza graphs.

Let Γ be a Deza graph with parameters (n, k, b, a), and let v be a vertex of Γ. Denote by $N(v)$ the set of neighbours of a vertex v, and let $\beta(v)$ be the number of vertices $u \in V(\Gamma)$ such that $|N(v) \cap N(u)| = b$. The number $\beta(v)$ does not depend on the choice of v and for $a \neq b$ is given by $\beta(v) = \beta = \frac{k(k-1)-a(n-1)}{b-a}$.

Strictly Deza graphs with parameters (n, k, b, a), where $k = b + 1$ and $\beta > 1$ are fulfilled, were investigated in [3]. Now we characterize strictly Deza graphs with parameters (n, k, b, a), where $k = b + 1$ and $\beta = 1$. Remark that divisible design graph is a special Deza graph [2], and a Deza graph with $\beta = 1$ is a divisible design graph.

References

[1] M. Erickson, S. Fernando, W. H. Haemers, D. Hardy, J. Hemmeter, *Deza graphs: A generalization of strongly regular graphs*, J. Comb. Designs. 7 (1999), 359–405.

[2] W. H. Haemers, H. Kharaghani, M. Meulenberg, *Divisible design graphs* J. Combinatorial Theory A, 118 (2011), 978–992.

[3] V. V. Kabanov, N. V. Maslova, L. V. Shalaginov, *On Deza graphs with parameters* $(v, k, k-1, a)$, arXiv:1712.09529v3.

Integral Cayley graphs

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A graph is *integral* if all eigenvalues of its adjacency matrix are integers. In 1974, F. Harary and A. J. Schwenk posed a question on graphs having integral spectra. Since the general problem of classifying integral graphs seems too difficult, special classes of graphs including trees, graphs of bounded degree, regular graphs and Cayley graphs are investigated. In a survey on integral graphs by K. Balińska, D. Cvetkovič, Z. Radosavljevič, S. Simič, and D. Stevanovič published in 2002 it was observed that the number of integral graphs is not only infinite, but one can find them among graphs of all orders. However, they are very rare and difficult to be found. In 2009, O. Ahmadi, N. Alon, I. F. Blake, and I. E. Shparlinski showed that most graphs have nonintegral eigenvalues.

Integrality of Cayley graphs was investigated by many researchers. The characterization of integral graphs among circulant graphs was done by W. So in 2005. Integral Cayley graphs over abelian groups were characterized in 2010 by W. Klotz and T. Sander. All connected cubic integral Cayley graphs were determined up to isomorphism by A. Abdollahi and E. Vatandoost in 2009 [1].

In this talk we discuss recent progress on integral Cayley graphs over finite groups and formulate related open problems. We show that the spectrum of a Cayley graph over a finite nilpotent group with a normal generating set S containing with every its element s all generators of the cyclic group $\langle s\rangle$ is integral. In particular, a Cayley graph of a 2-group generated by a normal set of involutions is integral. We prove that a Cayley graph over the symmetric group of degree $n \geqslant 2$ generated by all transpositions is integral. We find the spectrum of a Cayley graph over the alternating group of degree $n \geqslant 4$ with a generating set of 3-cycles of the form (kij) with fixed k, as

$$\{-n+1, 1-n+1, 2^2-n+1, \ldots, (n-1)^2-n+1\}.$$

This is joint paper with Daria Lytkina [3] supported by RFBR-18-01-00420.

References

[1] A. Abdollahi, E. Vatandoost, Which Cayley graphs are integral? *The Electronic Journal of Combinatorics*, **16** (2009) 6–7.

[2] F. Harary, A. J. Schwenk, Which graphs have integral spectra? *Graphs and Combinatorics*, **390** (1974) 45–51.

[3] D. V. Lytkina, E. V. Konstantinova, Integral Cayley graphs over finite groups, submitted to *Algebra Colloquium*.

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Dual Seidel switching and Deza graphs

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Permuting the rows (and not the columns) of the adjacency matrix of a graph is called *dual Seidel switching*. This operation was introduced in [H84] as a possible way to construct strongly regular graphs with $\lambda = \mu$ from the existing ones.

A k-regular graph G on v vertices is a *Deza graph* with parameters (v, k, b, a), where $v > k \geq b \geq a \geq 0$, if the number of common neighbours of two distinct vertices takes on one of two values a or b, not necessarily depending on the adjacency of the two vertices. A Deza graph is called a *strictly Deza graph* if it has diameter 2 and is not strongly regular. In [EFHHH99], the dual Seidel switching was adopted to obtain strictly Deza graphs from strongly regular graphs.

In this talk we give a survey of results on Deza graphs obtained by the dual Seidel switching.

References

[**H84**] Haemers W. H., Dual Seidel switching. Eindhoven: Technical University Eindhoven. 1984. P. 183–191.

[**EFHHH99**] Erickson M., Fernando S., Haemers W. H., Hardy D. and Hemmeter J., Deza graphs: a generalization of strongly regular graphs. J. Comb. Designs. 1999. V. 7. P. 359–405.

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[**GS13**] Goryainov S. V., Shalaginov L. V., On Deza graphs with triangular and lattice graph complements as parameters. J. Appl. Industr. Math. 2013. V. 3. P. 355–362.