# جلسه دفاع از رساله دکتری

# Differentiation in Logical Form

## مهرداد ملکی

### پژوهشگاه دانشهای بنیادی

چکیده

We introduce a logical theory of differentiation for a real-valued function on a finite dimensional real Euclidean space. A real-valued continuous function is represented by a localic approximable mapping between two semi-strong proximity lattices, representing the two coherent Euclidean spaces for the domain and the range of the function. Similarly, the Clarke subgradient, equivalently the L-derivative, of a locally Lipschitz map, which is non-empty, compact and convex valued, is represented by an approximable mapping. Corresponding to the notion of a single-tie of a locally Lipschitz function, used to derive the domain-theoretic L-derivative of the function, we introduce the dual notion of a single-knot of approximable mappings which gives rise to Lipschitzian approximable mappings. We then develop the notion of a strong single-tie and that of a strong knot leading to a Stone duality result for locally Lipschitz maps and Lipschitzian approximable mappings. The strong single-knots, in which a Lipschitzian approximable mapping belongs, are employed to define the Lipschitzian derivative of the approximable mapping. The latter is dual to the Clarke subgradient of the corresponding locally Lipschitz map defined domain- theoretically using strong single-ties. A stricter notion of strong single- knots is subsequently developed which captures approximable mappings of continuously differentiable maps providing a gradient Stone duality for these maps. We then derive a calculus for Lipschitzian derivative of approximable mapping for some basic constructors and show that it is dual to the calculus satisfied by the Clarke subgradiet. Finally we extend these results to the functions with imprecise input and complex maps.

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