

ON A QUESTION OF ZADROZNY

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In this short note, I will discuss the following question of Zadrozny [1].

Question 0.1. *Assume 0^\sharp does not exist. Let M be a model of ZFC. Is there a model N of ZFC extending M with $ON^N = ON^M$ such that $HOD^N = L$?*

Our next result gives a negative answer to this question in a strong way.

Theorem 0.2. *Any model V of ZFC has a class generic extension $V' \models \text{“ZFC”}$ such that if $W \supseteq V'$ is a model of ZFC with $ON^W = ON^{V'}$, then $HOD^W \neq V$.*

Proof. Force over V by the reverse Easton iteration to add a new Cohen subset to each successor cardinal. Call the resulting extension V_1 . By Jensen’s coding theorem, V_1 has a class generic extension V_2 such that for some real $R \in V_2$, we have $V_2 \models \text{“}V = L[R]\text{”}$. We show that $V' = V_2$ is as required. Thus suppose $W \supseteq V'$ is a model of ZFC, and suppose on the contrary that $HOD^W = V$. Then as $R \in W$, by a result of Vopenka, R is set generic over $HOD^W = V$, and this is a contradiction. \square

REFERENCES

- [1] Zadrozny, Wlodzimierz Iterating ordinal definability. *Ann. Pure Appl. Logic* 24 (1983), no. 3, 263-310.

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