On the birth of Set Theoretic Algebra

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Historical notes

Some Algebra

Some set theory

On the birth of Set theoretic Algebra

More historical notes

Whitehead's problem for W-groups of larger cardinality

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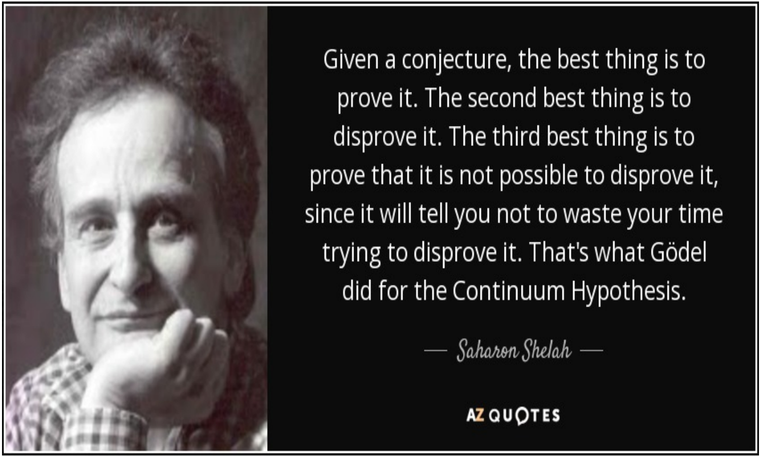

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Historical notes

- P. Eklof and A. Mekler, Almost Free Modules,North-Holland (1990); Preface begins:
- The modern era in set-theoretic methods in algebra can be said to have begun on July 11, 1973 when Saharon Shelah borrowed Laszlo Fuchs's Infinite Abelian Groups from the Hebrew University library. Soon thereafter, he showed that Whiteheads Problem, to which many talented mathematicians had devoted much creative energy, was not solvable in ordinary set theory (ZFC).

Historical notes

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Historical notes

- Whitehead's problem asks whether $Ext(A, \mathbb{Z}) = 0$ implies that A is free.
- In the 1950's, Stein and Ehrenfeucht showed that the answer is affirmative for countable A.
- For uncountable A prior to 1973, Nunke says: Many people: J. Rotman, myself, S. Chase, P. Griffith have studied this problem obtaining meager results.

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- Saharon Shelah relates that in 1973 he had the habit of looking every week at the new books displayed in the Hebrew University library.

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Historical notes

- Saharon Shelah relates that in 1973 he had the habit of looking every week at the new books displayed in the Hebrew University library.
- One day I have come and see the second volume of Laszlo; its colour was attractive green. I take it and ask myself isn't everything known on [abelian groups]... I start to read each linearly; after reading about two thirds of the first volume I move to the second volume and read the first third. I mark the problems (I think six) which attract me, combination of being stressed by Laszlo, seem to me I have a chance, and how nice the problem look.

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- By September 4, 1973, Shelah had submitted a paper, proving that the solution to Whitehead's problem is independent of ZFC and answering some other open problems as well.

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Historical notes

- By September 4, 1973, Shelah had submitted to the Israel Journal a paper, proving that the solution to Whitehead's problem is independent of ZFC and answering some other open problems as well.
- I have thought the most important is to build indecomposable abelian groups in every cardinality. I thought the independence of Whiteheads problem will be looked on suspiciously. As you know abelian group theorists thought differently [communication from Shelah].

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- Whitehead's problem asks:

Is every W-group free?

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ZFC axioms

- The underlying theory we consider is *ZFC*:

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ZFC axioms

- The underlying theory we consider is *ZFC*:
- *ZFC* =Ordinary Mathematics.
- But most of the talk goes beyond *ZFC*!!!.
- By Godel's theorems, *ZFC* is incomplete: there are statements which are neither provable nor refutable from *ZFC* axioms.
- The first such natural statement is the continuum hypothesis, which follows from the works of Godel (1938) and Paul Cohen (1963).

Going beyond *ZFC*

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- These axioms were used by Shelah to show that the Whitehead's problem is independent of *ZFC* for W-groups of size $\aleph_1$.
- We will then discuss the problem for W-groups of arbitrary size.

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- $C \subseteq \kappa$ is unbounded if $\forall \alpha < \kappa \exists \beta \in C (\alpha < \beta)$.

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- $C \subseteq \kappa$ is a club, if it is both closed and unbounded.
- $S \subseteq \kappa$ is stationary, if it intersects all club subsets of κ

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- $\diamondsuit_\kappa(S)$: There exists a sequence $(S_\alpha : \alpha < \kappa)$ such that:
    - $S_\alpha \subseteq \alpha$, for all $\alpha < \kappa$.
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- Godel's constructible universe L is the least transitive model of *ZFC* which contains all ordinals.
- (Ronald Jensen) In L, $\diamondsuit_{\kappa^+}(S)$ holds for all infinite cardinals κ and all stationary sets $S \subseteq \kappa^+$.

Going beyond *ZFC*

- A partially ordered set (poset) $\mathbb{P}$ satisfies the countable chain condition, if any antichain $A \subseteq \mathbb{P}$ is at most countable, where A is an antichain if for all $p, q \in A$ there is no $r \in \mathbb{P}$ with $r \leq p, q$.
- $G \subseteq \mathbb{P}$ is a filter, if
  - $G \neq \emptyset$.
  - If $p \in G$ and $p \leq q$, then $q \in G$.
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- $D \subseteq \mathbb{P}$ is dense if $\forall p \in \mathbb{P} \exists q \in D (q \leq p)$.
- Given a collection $\mathcal{D}$ of dense subsets of $\mathbb{P}$, $G \subseteq \mathbb{P}$ is called $\mathcal{D}$-generic filter if
  - G is a filter.
  - $G \cap D \neq \emptyset$, for all $D \in \mathcal{D}$.

Going beyond *ZFC*

- (Rasiowa-Sikorski) If $\mathbb{P}$ is a poset and $\mathcal{D}$ is a countable collection of dense subsets of $\mathbb{P}$, then there exists a $\mathcal{D}$-generic filter $G \subseteq \mathbb{P}$.

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- Martin's axiom at $\aleph_1$ $(MA_{\aleph_1})$ is the statement: If $\mathbb{P}$ is a poset that satisfies the countable chain condition and if $\mathcal{D}$ is a collection of dense subsets of $\mathbb{P}$ of size $\aleph_1$, then there exists a $\mathcal{D}$-generic filter $G \subseteq \mathbb{P}$.

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- (Solovay-Tennenbaum) $MA_{\aleph_1} + 2^{\aleph_0} = \aleph_2$ is consistent with *ZFC*.

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- Assume $MA_{\aleph_1} + 2^{\aleph_0} = \aleph_2$. Then there exists a W-group of size $\aleph_1$ which is not free.
- Thus Whitehead's problem for W-groups of size $\aleph_1$ is independent of ZFC.

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- Shelah's result left open the question of whether the continuum hypothesis (CH) was sufficient to imply that W-groups are free.

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- Shelah says "Naturally, it was hoped that [the fact that W-groups are free] was not a consequence of CH alone.

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- Shelah says "Naturally, it was hoped that [the fact that W-groups are free] was not a consequence of CH alone.
- But some algebraists hoped otherwise, and, in fact, some false proofs from CH circulated.
- In Math. Reviews, a review of L. Fuchs' Infinite Abelian Groups, vol II states: Since Volume II was written, S. Shelah [Israel J. Math. 18 (1974), 243-256] has shown that the statement Every W-group of cardinal $\aleph_1$ is free is independent of ZFC (Paul Hill has shown that GCH implies that every W-group of cardinality $\aleph_\Omega$ is free and J. Rotman has an easy proof that CH implies that every W-group is free).

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- In 1977, Shelah proved the following theorem:

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- It is consistent with $ZFC + GCH$ that there are Whitehead groups of cardinality $\aleph_1$ which are not free.

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- In 1977, Shelah proved the following theorem:
- It is consistent with $ZFC + GCH$ that there are Whitehead groups of cardinality $\aleph_1$ which are not free.
- Shelah's theorem shows that the above cited claims are false.

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Shelah's Singular compactness theorem

- Shelah's proof of the consistency of Whitehaed's problem, in fact gives the following:

Shelah's Singular compactness theorem

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- By a theorem of Hill, if κ is a singular cardinal of cofinality ω, or ω_1 and if A is a group of size κ all of whose subgroups of size $< \kappa$ are free, then A is free.

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- By a theorem of Hill, if κ is a singular cardinal of cofinality ω, or ω_1 and if A is a group of size κ all of whose subgroups of size $< \kappa$ are free, then A is free.
- This allows us to show that in the Godel's universe, all W-groups of size $< \aleph_{\omega_2}$ are free.

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- After studying the paper, Shelah was able to prove a very general "Singular Compactness Theorem".
- It applies to an abstract notion of "free"-defined axiomatically-and says, roughly, that if an object of singular cardinality κ has the property that "most" of its subobjects of cardinality $< \kappa$ are "free", then the object is "free". In particular, the theorem applies to the standard notion of "free" in any variety.

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Shelah's Singular compactness theorem

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- As a consequence, in Godel's universe, every Whitehead group (of arbitrary cardinality) is free.
- Putting all the above results together, we get the following independence result:

Theorem (Shelah):

Whitehead's problem is independent of *ZFC*.

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Thank You for Your Attension

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