Three variable full orthogonal designs of order 56

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Abstract

Using the doubling lemma, amicable sets of eight circulant matrices and four complementary negacyclic matrices, we show that all full triples are types of orthogonal designs of order 56. This implies that all full orthogonal designs $\mathrm{OD}(2^t 7; x, y,\, 2^t 7 - x - y)$ exist for any $t \geq 3$.

Keywords: Orthogonal designs, amicable sets of matrices, negacyclic matrices.
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1 Introduction

An *orthogonal design* A of order n and type $(s_1, s_2, \ldots, s_u)$, in the commuting variables $\pm x_1, \pm x_2, \ldots, \pm x_u$, denoted $\mathrm{OD}(n; s_1, s_2, \ldots, s_u)$, is a square matrix of order n with entries $\pm x_k$ or 0, where each x_k occurs s_k times in each row and column such that distinct rows are pairwise orthogonal. In other words,

$$AA^T = (s_1 x_1^2 + \cdots + s_u x_u^2) I_n,$$

where I_n is the identity matrix of order n. It is known that the maximum number of variables in an orthogonal design is $\rho(n)$, the Radon number, defined as $\rho(n) = 8c + 2^d$, where $n = 2^a b$, b odd, $a = 4c + d$ and $0 \leq d < 4$.

It is conjectured that all full 3-tuples are types of orthogonal designs of order $8n$, see [3]. This conjecture has been verified for $n \leq 6$ [3, 5, 7]. In a recent paper [2], a number of full orthogonal designs of order 56 are found and 82 triples are labeled as missing. In this paper, we will show that the conjecture also holds for $n = 7$. Our approach is to first generate all possible full triples in order 56 using the doubling lemma. To do so, we apply the doubling lemma to all possible sets of four complementary circulant matrices of order seven in four variables. Then we search to find as many as possible sets of amicable circulant matrices of order 7 in eight or less variables and identify all those which are special amicable. In doing so, we are able to resolve a total of 224 triples out of 261 possible full triples. While the use of circulant matrices in constructing orthogonal designs is standard, it was shown in [7, 8] that negacyclic matrices may be used for those cases where the circulant matrices either do not exist or are hard to find. We will use this method to construct the 37 missing orthogonal designs in order 56. We have the following results.

Theorem 1 *All* 261 *possible full orthogonal designs of order 56 in three variables exist.*

Corollary 1 *All full orthogonal designs* $OD(2^t 7; x, y,\ 2^t 7 - x - y)$ *exist for any* $t \geq 3$.

The complete set of 261 orthogonal designs of order 56 in three variables is available at the website http://www.cs.uleth.ca/OD56triples.

[Note to the referees: the password is coolcow]

2 Amicable sets of circulant matrices

Two orthogonal designs X and Y are called *amicable* if $XY^t = YX^t$. A set $\{A_1, A_2, \ldots, A_{2n}\}$ of square real matrices is said to be *amicable* if

$$\sum_{i=1}^{n} (A_{\sigma(2i-1)} A^t_{\sigma(2i)} - A_{\sigma(2i)} A^t_{\sigma(2i-1)}) = 0,$$

for some permutation σ of the set $\{1, 2, \ldots, 2n\}$. In this case, we say that $\{A_{\sigma(2i-1)}\}_{i=1}^{n}$ *matches* with $\{A_{\sigma(2i)}\}_{i=1}^{n}$. For simplicity, we will always take $\sigma(i) = i$ unless otherwise specified. Clearly, a set of mutually amicable matrices is amicable, but the converse is not true in general. A set of matrices $\{A_1, A_2, \ldots, A_n\}$ is said to satisfy an additive property, if the matrix $\sum_{i=1}^{n} A_i A_i^t$ is a multiple of the identity matrix.

An amicable set of eight circulant matrices $\{A_1, A_2, \ldots, A_8\}$ of order n satisfying an additive property can be used in the Kharaghani array [9]:

$$K = \begin{pmatrix} A_1 & A_2 & A_4R & A_3R & A_6R & A_5R & A_8R & A_7R \\ -A_2 & A_1 & A_3R & -A_4R & A_5R & -A_6R & A_7R & -A_8R \\ -A_4R & -A_3R & A_1 & A_2 & -A_8^tR & A_7^tR & A_6^tR & -A_5^tR \\ -A_3R & A_4R & -A_2 & A_1 & A_7^tR & A_8^tR & -A_5^tR & -A_6^tR \\ -A_6R & -A_5R & A_8^tR & -A_7^tR & A_1 & A_2 & -A_4^tR & A_3^tR \\ -A_5R & A_6R & -A_7^tR & -A_8^tR & -A_2 & A_1 & A_3^tR & A_4^tR \\ -A_8R & -A_7R & -A_6^tR & A_5^tR & A_4^tR & -A_3^tR & A_1 & A_2 \\ -A_7R & A_8R & A_5^tR & A_6^tR & -A_3^tR & -A_4^tR & -A_2 & A_1 \end{pmatrix},$$

where R is the back–identity matrix of order n, to obtain an orthogonal matrix of order $8n$.

In order to find orthogonal designs of order 56 using the array K above, we need to search for amicable sets of eight circulant matrices of order 7 satisfying an additive property. We began our search from the known sets of four circulant matrices of order 7 in more than one variable taken from [2, 3]. The first rows of these matrices are listed in Table 1. In our search we found it convenient to use additional sets for some tuples, which are listed at the bottom of Table 1. They are sets of four matrices of order 7 in at most four variables satisfying an additive property. Then we changed the variables to new variables thereby obtaining another list of four circulant matrices. Now, the main stage in the search is to find all possible amicable sets of eight matrices, where four matrices belongs to one list and the rest to the other. In order to obtain an optimal number of solutions, the implementation of this procedure includes an appropriate collapsing of variables [5]. By applying this method, we are able to find 209 types of three variable full orthogonal designs in order 56.

An amicable set which is obtained by a matching between the two lists is called a *special amicable set* [6]. Each of the special amicable sets of matrices can be used in other arrays to generate infinitely many orthogonal designs, see [6, 9].

We present in Table 3 some amicable sets found in the searches. From these 81 amicable sets, amicable sets for 209 triples can be found by equating variables. If the matching given is special, then the set is flagged with an asterisk. There are 61 special sets.

A file at the website http://www.cs.uleth.ca/OD56triples lists amicable sets for each of the 209 triples found in the searches. We found special amicable sets for

194 of them, leaving just 15 for which no special matching is found yet. Please note that the number of special amicable sets is higher than the number implied from Table 3 because special sets may exist for triples even though none exists prior to the equating of variables.

Table 1: Known OD of order 28

type	A	B	C	D
$(1,1,1,25)$	$(a,d,d,\bar{d},d,\bar{d},\bar{d})$	$(b,d,d,\bar{d},d,\bar{d},\bar{d})$	$(c,d,d,\bar{d},d,\bar{d},\bar{d})$	$(\bar{d},d,d,d,d,d,d)$
$(1,1,8,18)$	$(\bar{a},b,b,a,b,a,a)$	$(a,b,b,\bar{a},b,\bar{a},\bar{a})$	$(c,b,b,\bar{b},b,\bar{b},\bar{b})$	$(d,b,b,\bar{b},b,\bar{b},\bar{b})$
$(1,1,13,13)$	$(\bar{a},b,b,a,b,a,a)$	$(\bar{b},\bar{a},\bar{a},b,\bar{a},b,b)$	$(c,a,a,\bar{a},a,\bar{a},\bar{a})$	$(d,b,b,\bar{b},b,\bar{b},\bar{b})$
$(1,3,6,18)$	$(\bar{d},c,\bar{c},d,d,a,\bar{d})$	$(d,b,\bar{c},\bar{c},\bar{d},d,\bar{d})$	$(d,c,\bar{d},b,\bar{d},\bar{d},\bar{d})$	$(c,b,d,d,\bar{d},d,d)$
$(4,4,4,16)$	$(c,d,\bar{c},c,a,c,b)$	$(c,d,\bar{c},c,\bar{a},c,\bar{b})$	$(c,d,\bar{c},\bar{c},a,\bar{c},\bar{b})$	$(c,d,\bar{c},\bar{c},\bar{a},\bar{c},b)$
$(4,4,10,10)$	$(b,c,a,c,d,d,\bar{d})$	$(b,\bar{c},a,\bar{c},\bar{d},\bar{d},d)$	$(b,d,\bar{a},d,\bar{c},\bar{c},c)$	$(b,\bar{d},\bar{a},\bar{d},c,c,\bar{c})$
$(7,7,7,7)$	$(a,a,\bar{a},b,c,b,d)$	$(\bar{b},\bar{b},b,a,d,a,\bar{c})$	$(\bar{c},\bar{c},c,\bar{d},a,\bar{d},b)$	$(\bar{d},\bar{d},d,c,\bar{b},c,a)$
$(1,3,24)$	$(a,c,\bar{c},c,\bar{c},c,\bar{c})$	$(b,c,\bar{c},\bar{c},\bar{c},\bar{c},\bar{c})$	$(b,\bar{c},\bar{c},c,c,c,c)$	$(b,\bar{c},c,c,\bar{c},c,c)$
$(1,6,21)$	$(c,b,\bar{b},b,\bar{b},b,\bar{b})$	$(a,b,b,b,b,\bar{b},\bar{b})$	$(a,\bar{a},b,b,\bar{a},b,b)$	$(a,a,b,\bar{b},\bar{b},b,b)$
$(2,4,22)$	$(c,\bar{c},\bar{c},c,c,b,\bar{a})$	$(\bar{c},c,\bar{c},c,a,b,c)$	$(c,\bar{c},c,c,c,\bar{b},c)$	$(\bar{c},c,c,c,c,b,\bar{c})$
$(2,7,19)$	$(c,a,a,a,\bar{a},b,\bar{a})$	$(\bar{c},a,b,a,\bar{a},a,\bar{a})$	$(b,\bar{b},a,a,\bar{b},a,a)$	$(b,b,a,\bar{a},\bar{a},a,a)$
$(2,8,18)$	$(b,c,c,a,\bar{c},\bar{c},b)$	$(\bar{b},\bar{c},c,a,\bar{c},c,\bar{b})$	$(b,c,\bar{c},c,c,c,\bar{b})$	$(b,c,c,c,\bar{c},c,\bar{b})$
$(2,9,17)$	$(a,\bar{b},c,\bar{b},b,c,b)$	$(a,c,\bar{c},c,\bar{c},\bar{c},\bar{c})$	$(b,b,\bar{c},c,b,\bar{c},c)$	$(\bar{b},\bar{c},b,c,c,c,c)$
$(3,10,15)$	$(a,\bar{b},b,b,c,b,c)$	$(a,b,\bar{b},\bar{b},c,\bar{c},\bar{c})$	$(a,\bar{c},\bar{b},\bar{c},\bar{c},c,c)$	$(b,\bar{c},b,\bar{c},c,\bar{c},\bar{c})$
$(5,5,18)$	$(a,b,\bar{c},b,c,\bar{c},c)$	$(a,\bar{c},a,c,b,\bar{b},\bar{b})$	$(a,\bar{c},\bar{a},\bar{c},\bar{c},c,\bar{c})$	$(\bar{c},\bar{c},c,c,c,c,c)$
$(5,9,14)$	$(a,\bar{b},b,\bar{c},c,c,c)$	$(a,b,\bar{c},a,c,\bar{b},\bar{c})$	$(b,\bar{b},\bar{b},\bar{b},\bar{c},\bar{b},c)$	$(\bar{a},c,c,a,c,\bar{c},c)$
$(7,8,13)$	$(\bar{c},b,\bar{a},a,a,b,\bar{c})$	$(\bar{a},c,b,\bar{c},c,b,c)$	$(a,\bar{c},\bar{b},c,c,b,c)$	$(\bar{c},b,a,c,a,\bar{b},c)$
$(8,9,11)$	$(\bar{c},b,\bar{b},a,c,\bar{c},a)$	$(b,a,a,c,a,\bar{a},\bar{b})$	$(c,\bar{a},b,b,b,\bar{c},a)$	$(\bar{c},c,c,c,b,c,\bar{b})$
$(9,9,10)$	$(a,a,a,\bar{b},b,\bar{c},c)$	$(\bar{b},b,c,b,b,b,\bar{c})$	$(a,b,\bar{a},\bar{b},c,\bar{c},\bar{c})$	$(\bar{a},a,\bar{a},c,a,c,c)$
$(5,23)$	$(a,a,\bar{a},\bar{b},b,b,b)$	$(\bar{a},b,\bar{a},b,\bar{b},b,\bar{b})$	$(b,b,b,b,b,\bar{b},\bar{b})$	$(b,b,b,\bar{b},b,b,\bar{b})$
$(11,17)$	$(b,\bar{b},b,b,b,a,\bar{a})$	$(b,\bar{b},\bar{b},\bar{b},b,\bar{a},\bar{a})$	$(b,a,\bar{a},\bar{a},\bar{a},\bar{a},a)$	$(a,a,a,\bar{a},a,a,\bar{a})$
$(7,7,7,7)$	$(\bar{a},a,a,b,a,c,d)$	$(\bar{b},b,b,\bar{a},b,d,\bar{c})$	$(\bar{c},c,c,\bar{d},c,\bar{a},b)$	$(\bar{d},d,d,c,d,\bar{b},\bar{a})$
$(5,23)$	$(b,\bar{b},\bar{a},a,a,b,b)$	$(\bar{b},\bar{b},a,\bar{b},a,b,b)$	$(b,b,\bar{b},b,\bar{b},b,b)$	$(\bar{b},b,b,b,\bar{b},b,b)$
$(11,17)$	$(\bar{a},a,a,\bar{b},b,b,b)$	$(b,a,\bar{b},\bar{b},\bar{a},a,\bar{b})$	$(\bar{b},b,a,a,b,a,\bar{b})$	$(a,b,\bar{b},b,\bar{a},b,b)$

3 The doubling lemma

Some orthogonal designs of order 56 can be constructed using the known orthogonal designs in order 28. The following doubling lemma is especially useful.

Lemma 1 [3, page 79] *If there is an orthogonal design in order* n *and of type* $(s_1, s_2, \dots, s_u)$*, then there is an orthogonal design in order* $2n$ *and of type* $(s_1, s_1, 2s_2, \dots, 2s_u)$.

Using the doubling lemma on the orthogonal designs of order 28 given in Table 1, 15 three variable full orthogonal designs of order 56 were constructed. The types of these designs are listed in Table 2, where the variable weights that are repeated, corresponding to s_1 in the statement of Lemma 1, are flagged with asterisks. When more than one weight is flagged, the corresponding variables are set equal before applying the doubling lemma. After the doubling lemma is applied, additional variables are set equal to finally obtain the desired types of three variable orthogonal designs of order 56.

Table 2: Types of OD in order 56 obtained by the doubling lemma

Order 56 type obtained	Order 28 type used
$(1, 12, 43)$	$(1^*, 6, 21)$
$(3, 23, 30)$	$(3^*, 10, 15)$
$(4, 17, 35)$	$(2, 9, 17^*)$
$(5, 10, 41)$	$(5^*, 5, 18)$
$(5, 12, 39)$	$(1, 3^*, 6, 18)$
$(5, 15, 36)$	$(5^*, 5, 18)$
$(5, 18, 33)$	$(5^*, 9, 14)$
$(6, 10, 40)$	$(3, 10^*, 15)$
$(6, 13, 37)$	$(1^*, 3, 6, 18)$
$(6, 15, 35)$	$(3, 10, 15^*)$
$(7, 12, 37)$	$(1^*, 3, 6, 18)$
$(9, 13, 34)$	$(2, 9^*, 17)$
$(12, 21, 23)$	$(1, 6, 21^*)$
$(16, 19, 21)$	$(1^*, 1, 8, 18^*)$
$(17, 18, 21)$	$(2, 9, 17^*)$

4 Construction using negacyclic matrices

A negacyclic matrix of order n is any polynomial in the negashift matrix U of order n, where $U = [u_{ij}]$, $u_{i(i+1)} = 1$, $i = 1, 2, \cdots, n-1$, $u_{n1} = -1$, and $u_{ij} = 0$ if $j - i \not\equiv 1 \pmod{n}$. The class of negacyclic matrices of order n form a commutative subring of the ring of all matrices of order n, see [1, 3] for details.

It is known that if A_1, A_2, A_3, A_4 are four circulant or four negacyclic matrices of order n satisfying an additive property, then upon substitution in the Goethals-Seidel array

$$G = \begin{pmatrix} A_1 & A_2R & A_3R & A_4R \\ -A_2R & A_1 & A_4^tR & -A_3^tR \\ -A_3R & -A_4^tR & A_1 & A_2^tR \\ -A_4R & A_3^tR & -A_2^tR & A_1 \end{pmatrix},$$

one finds an orthogonal matrix of order $4n$. Note that the matrix R is the back–diagonal identity of order n. It is a standard practice to use this array in constructing orthogonal designs. We have two options for the type of matrices, namely, circulant or negacyclic. Our experience from our previous works had indicated that using negacyclic matrices were always more fruitful.

We did, as explained in Section 2, a thorough search for amicable sets of eight matrices and this resulted in 209 triples of order 56. An additional 14 were obtained via the doubling lemma in Section 3. Since there are 261 possible triples, we are left with 37 triples yet to be found. Therefore, we searched for sets of four negacyclic matrices of order 14 in three variables satisfying appropriate additive properties and found all of the remaining orthogonal designs. These are listed in Table 4. We now describe our search method briefly. The search consisted of three steps. In the first step we generated and saved all possible negacyclic $(0, \pm 1)$-matrices of order 14 and classified them by looking at the inner products of pairs of rows. In this way we found 80217 classes of matrices. This number of classes was too large to be dealt with and we restricted the values obtained from the inner products of pairs of rows to $0, \pm 1$. This reduced the number of classes to 1227. In the second step, for any $1 \leq s \leq 56$, we found all weighing matrices $W(56, s)$, constructible from 4 negacyclic matrices belonging to the classes which were retained in step one, via Goethals-Seidel array. In the last step, for finding an $\mathrm{OD}(56, s_1, s_2, 56 - s_1 - s_2)$, we searched for disjoint weighing matrices $W_1 = W(56, s_1)$, $W_2 = W(56, s_2)$ and $W_3 = W(56, 56 - s_1 - s_2)$ among those found in the second step and tested if $aW_1 + bW_2 + cW_3$ was an orthogonal design in variables a, b, c. It is notable that the computations required for different types of orthogonal designs ranged from a few seconds to a couple of days on a single 2.6 GHz PC.

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Table 3: Amicable sets for given OD of order 56

type	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$(4_4,10_4)^*$	$aebef\bar{f}\bar{f}$	$cgdghh\bar{h}$	$a\bar{e}b\bar{e}\bar{f}\bar{f}f$	$c\bar{g}d\bar{g}\bar{h}\bar{h}h$	$af\bar{b}f\bar{e}\bar{e}e$	$c\bar{h}\bar{d}\bar{h}gg\bar{g}$	$a\bar{f}\bar{b}\bar{f}ee\bar{e}$	$ch\bar{d}h\bar{g}\bar{g}g$
$(7_8)^*$	$\bar{c}ccacbd$	$\bar{g}ggegfh$	$\bar{a}aa\bar{c}ad\bar{b}$	$e\bar{e}\bar{e}g\bar{e}hf$	$\bar{b}bb\bar{d}b\bar{c}a$	$h\bar{h}\bar{h}\bar{f}\bar{h}eg$	$\bar{d}ddbd\bar{a}\bar{c}$	$f\bar{f}\bar{f}h\bar{f}g\bar{e}$
$(1_2,2_2,25_2)^*$	$cee\bar{e}e\bar{e}\bar{e}$	$dff\bar{f}f\bar{f}\bar{f}$	$aee\bar{e}e\bar{e}\bar{e}$	$f\bar{f}\bar{f}\bar{f}\bar{f}\bar{f}\bar{f}$	$cee\bar{e}e\bar{e}\bar{e}$	$\bar{d}\bar{f}\bar{f}f\bar{f}ff$	$\bar{e}eeeeee$	$\bar{b}\bar{f}\bar{f}f\bar{f}ff$
$(2_2,8_2,18_2)^*$	$\bar{c}eececc$	$\bar{d}ffdfdd$	$cee\bar{c}e\bar{c}\bar{c}$	$dff\bar{d}f\bar{d}\bar{d}$	$aee\bar{e}e\bar{e}\bar{e}$	$\bar{b}\bar{f}\bar{f}f\bar{f}ff$	$aee\bar{e}e\bar{e}\bar{e}$	$bff\bar{f}f\bar{f}\bar{f}$
$(4_4,8,32)^*$	$fe\bar{f}fbfa$	$fe\bar{f}fdfc$	$fe\bar{f}f\bar{b}f\bar{a}$	$fe\bar{f}f\bar{d}f\bar{c}$	$fe\bar{f}\bar{f}b\bar{f}\bar{a}$	$fe\bar{f}\bar{f}d\bar{f}\bar{c}$	$fe\bar{f}\bar{f}\bar{b}\bar{f}a$	$fe\bar{f}\bar{f}\bar{d}\bar{f}c$
$(1_3,25,28)^*$	$cdd\bar{d}d\bar{d}\bar{d}$	$ee\bar{e}ee\bar{e}e$	$add\bar{d}d\bar{d}\bar{d}$	$e\bar{e}eeee\bar{e}$	$bdd\bar{d}d\bar{d}\bar{d}$	$eee\bar{e}\bar{e}ee$	$\bar{d}dddddd$	$e\bar{e}\bar{e}e\bar{e}ee$
$(1_2,2,26_2)^*$	$bdd\bar{d}d\bar{d}\bar{d}$	$ee\bar{e}ee\bar{e}e$	$add\bar{d}d\bar{d}\bar{d}$	$e\bar{e}eeee\bar{e}$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{e}\bar{c}e\bar{e}eee$	$\bar{d}dddddd$	$eeec\bar{e}\bar{e}e$
$(1,3,8,19,25)^*$	$bee\bar{e}e\bar{e}\bar{e}$	$\bar{c}ddcdcc$	$bee\bar{e}e\bar{e}\bar{e}$	$cdd\bar{c}d\bar{c}\bar{c}$	$bee\bar{e}e\bar{e}\bar{e}$	$\bar{d}\bar{d}\bar{d}d\bar{d}dd$	$\bar{e}eeeeee$	$add\bar{d}d\bar{d}\bar{d}$
$(1,3,13,14,25)^*$	$bee\bar{e}e\bar{e}\bar{e}$	$\bar{c}ddcdcc$	$bee\bar{e}e\bar{e}\bar{e}$	$dcc\bar{d}c\bar{d}\bar{d}$	$bee\bar{e}e\bar{e}\bar{e}$	$\bar{d}\bar{d}\bar{d}d\bar{d}dd$	$\bar{e}eeeeee$	$acc\bar{c}c\bar{c}\bar{c}$
$(3,4,8,16,25)^*$	$dc\bar{d}dcdb$	$\bar{e}\bar{e}\bar{e}\bar{e}eea$	$dc\bar{d}d\bar{c}d\bar{b}$	$\bar{e}\bar{e}e\bar{e}\bar{e}ae$	$dc\bar{d}\bar{d}c\bar{d}\bar{b}$	$e\bar{e}\bar{e}e\bar{e}e\bar{e}$	$dc\bar{d}\bar{d}\bar{c}db$	$\bar{a}e\bar{e}\bar{e}\bar{e}\bar{e}\bar{e}$
$(4,7_2,14,24)^*$	$ee\bar{e}eaee$	$\bar{c}\bar{c}cb\bar{d}bd$	$ee\bar{e}e\bar{a}e\bar{e}$	$\bar{d}\bar{d}d\bar{d}\bar{b}\bar{d}\bar{c}$	$ee\bar{e}\bar{e}a\bar{e}\bar{e}$	$\bar{d}\bar{d}ddcd\bar{b}$	$ee\bar{e}\bar{e}\bar{a}\bar{e}e$	$\bar{b}\bar{b}b\bar{c}d\bar{c}d$
$(1,1,18,36)$	$\bar{d}ccdcdd$	$dcc\bar{d}c\bar{d}\bar{d}$	$dd\bar{d}dd\bar{d}d$	$acc\bar{c}c\bar{c}\bar{c}$	$\bar{c}c\bar{c}\bar{c}bcc$	$dd\bar{d}d\bar{d}d\bar{d}$	$\bar{d}\bar{d}ddddd$	$dd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$
$(1,2,4,49)$	$bdd\bar{d}d\bar{d}\bar{d}$	$bdd\bar{d}d\bar{d}\bar{d}$	$\bar{d}dddddd$	$add\bar{d}d\bar{d}\bar{d}$	$dd\bar{d}dcdd$	$dd\bar{d}d\bar{c}d\bar{d}$	$dd\bar{d}\bar{d}\bar{c}dd$	$dd\bar{d}dc\bar{d}\bar{d}$
$(1,2,8,45)^*$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{c}ddcdcc$	$ddd\bar{d}d\bar{d}\bar{d}$	$cdd\bar{c}d\bar{c}\bar{c}$	$bdd\bar{d}d\bar{d}\bar{d}$	$bdd\bar{d}d\bar{d}\bar{d}$	$\bar{d}dddddd$	$add\bar{d}d\bar{d}\bar{d}$
$(1,2,13,40)^*$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{c}ddcdcc$	$ddd\bar{d}d\bar{d}\bar{d}$	$dcc\bar{d}c\bar{d}\bar{d}$	$bdd\bar{d}d\bar{d}\bar{d}$	$bdd\bar{d}d\bar{d}\bar{d}$	$\bar{d}dddddd$	$acc\bar{c}c\bar{c}\bar{c}$
$(1,2,14,39)$	$cdd\bar{d}d\bar{d}\bar{d}$	$dcc\bar{c}c\bar{c}\bar{c}$	$bdd\bar{d}d\bar{d}\bar{d}$	$bdd\bar{d}d\bar{d}\bar{d}$	$add\bar{d}d\bar{d}\bar{d}$	$\bar{d}dddddd$	$\bar{d}\bar{c}\bar{c}d\bar{c}dd$	$\bar{c}ddcdcc$
$(1,2,18,35)$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{d}ccdcdd$	$dcc\bar{d}c\bar{d}\bar{d}$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{d}dddddd$	$add\bar{d}d\bar{d}\bar{d}$	$bcc\bar{c}c\bar{c}\bar{c}$	$bcc\bar{c}c\bar{c}\bar{c}$
$(1,2,19,34)$	$ddd\bar{d}d\bar{d}\bar{d}$	$ccc\bar{c}c\bar{c}\bar{c}$	$bdd\bar{d}d\bar{d}\bar{d}$	$bdd\bar{d}d\bar{d}\bar{d}$	$acc\bar{c}c\bar{c}\bar{c}$	$\bar{d}dddddd$	$\bar{d}ccdcdd$	$dcc\bar{d}c\bar{d}\bar{d}$
$(1,3,24,28)^*$	$d\bar{d}ddddd$	$cb\bar{c}\bar{c}ccc$	$dd\bar{d}d\bar{d}d\bar{d}$	$c\bar{c}\bar{c}bc\bar{c}\bar{c}$	$dd\bar{d}\bar{d}d\bar{d}\bar{d}$	$bc\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}$	$dd\bar{d}\bar{d}\bar{d}\bar{d}d$	$\bar{a}\bar{c}c\bar{c}c\bar{c}c$
$(1,5,23,27)^*$	$ad\bar{d}d\bar{d}d\bar{d}$	$ccc\bar{c}\bar{c}cc$	$dd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$c\bar{b}c\bar{b}\bar{c}\bar{c}c$	$d\bar{d}\bar{d}dddd$	$bc\bar{c}cc\bar{c}c$	$d\bar{d}dd\bar{d}dd$	$cbcc\bar{c}c\bar{b}$
$(1,6,21,28)^*$	$ac\bar{c}c\bar{c}c\bar{c}$	$\bar{d}dd\bar{d}\bar{d}dd$	$bcccc\bar{c}\bar{c}$	$\bar{d}d\bar{d}dd\bar{d}d$	$b\bar{b}cc\bar{b}cc$	$\bar{d}\bar{d}dddd\bar{d}$	$bbc\bar{c}\bar{c}cc$	$\bar{d}dddddd$
$(1,7,21,27)^*$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{c}cccc\bar{c}\bar{b}$	$add\bar{d}d\bar{d}\bar{d}$	$b\bar{c}c\bar{c}\bar{c}c\bar{c}$	$\bar{d}ddd\bar{d}d\bar{d}$	$b\bar{b}\bar{b}\bar{c}\bar{b}\bar{c}\bar{c}$	$\bar{d}dddddd$	$\bar{c}cc\bar{c}\bar{c}b\bar{c}$
$(1,8,20,27)^*$	$ad\bar{d}d\bar{d}d\bar{d}$	$ccc\bar{c}\bar{c}cc$	$dd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$bc\bar{c}\bar{b}\bar{b}\bar{c}\bar{b}$	$d\bar{d}\bar{d}dddd$	$c\bar{c}bc\bar{b}cc$	$d\bar{d}dd\bar{d}dd$	$\bar{c}c\bar{c}bc\bar{c}b$
$(1,9,18,28)^*$	$\bar{b}ccbcbb$	$ddd\bar{d}d\bar{d}\bar{d}$	$bcc\bar{b}c\bar{b}\bar{b}$	$\bar{d}dd\bar{d}d\bar{d}\bar{d}$	$cc\bar{c}c\bar{c}\bar{c}a$	$dddddd\bar{d}$	$bcc\bar{c}c\bar{c}\bar{c}$	$d\bar{d}\bar{d}d\bar{d}\bar{d}d$
$(1,9,19,27)^*$	$ad\bar{d}d\bar{d}d\bar{d}$	$ccc\bar{c}\bar{c}cc$	$dd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$\bar{b}b\bar{c}\bar{b}\bar{c}\bar{c}b$	$d\bar{d}\bar{d}dddd$	$bbb\bar{c}c\bar{c}c$	$d\bar{d}dd\bar{d}dd$	$\bar{b}\bar{c}bc\bar{c}cc$
$(1,10,18,27)^*$	$ad\bar{d}d\bar{d}d\bar{d}$	$ccc\bar{c}\bar{c}cc$	$dd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$bb\bar{c}bc\bar{c}c$	$d\bar{d}\bar{d}dddd$	$c\bar{b}\bar{c}\bar{b}bb\bar{b}$	$d\bar{d}dd\bar{d}dd$	$cbcc\bar{c}c\bar{b}$
$(1,14,14,27)$	$add\bar{d}d\bar{d}\bar{d}$	$\bar{d}dddddd$	$bdd\bar{d}d\bar{d}\bar{d}$	$dbb\bar{b}b\bar{b}\bar{b}$	$cdd\bar{d}d\bar{d}\bar{d}$	$dcc\bar{c}c\bar{c}\bar{c}$	$\bar{b}\bar{c}\bar{c}b\bar{c}bb$	$\bar{c}bbcbcc$
$(2,2,14,38)^*$	$addd\bar{d}c\bar{d}$	$ccd\bar{d}\bar{d}dd$	$\bar{a}dcd\bar{d}d\bar{d}$	$dc\bar{c}dd\bar{c}d$	$c\bar{c}dd\bar{c}dd$	$dcd\bar{d}d\bar{d}\bar{b}$	$ccd\bar{d}\bar{d}dd$	$bddd\bar{d}c\bar{d}$
$(2,3,3,48)^*$	$ad\bar{d}d\bar{d}d\bar{d}$	$ad\bar{d}d\bar{d}d\bar{d}$	$cd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$bd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$dc\bar{d}dd\bar{d}d$	$b\bar{d}\bar{d}dddd$	$\bar{d}\bar{d}ddddc$	$b\bar{d}dd\bar{d}\bar{d}d$
$(2,4,12,38)$	$\bar{c}ddcdcc$	$cdd\bar{c}d\bar{c}\bar{c}$	$add\bar{d}d\bar{d}\bar{d}$	$add\bar{d}d\bar{d}\bar{d}$	$dd\bar{d}dbdc$	$dd\bar{d}d\bar{b}d\bar{c}$	$dd\bar{d}\bar{d}\bar{b}\bar{d}c$	$dd\bar{d}\bar{d}b\bar{d}\bar{c}$
$(2,4,22,28)^*$	$dd\bar{d}d\bar{d}d\bar{d}$	$cccb\bar{c}\bar{c}c$	$dd\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$c\bar{c}cabc\bar{c}$	$d\bar{d}\bar{d}dddd$	$c\bar{c}\bar{c}ccb\bar{a}$	$d\bar{d}dd\bar{d}dd$	$c\bar{b}cc\bar{c}cc$
$(2,4,24,26)^*$	$ccbccc\bar{c}$	$ddddd\bar{d}\bar{d}$	$c\bar{c}b\bar{c}\bar{c}\bar{c}c$	$da\bar{d}\bar{d}\bar{d}d\bar{d}$	$c\bar{c}\bar{c}ccc\bar{b}$	$\bar{a}\bar{d}\bar{d}\bar{d}d\bar{d}d$	$c\bar{c}\bar{b}\bar{c}cc\bar{c}$	$ddd\bar{d}d\bar{d}\bar{d}$
$(2,5,23,26)$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{b}\bar{b}c\bar{c}ccb$	$add\bar{d}d\bar{d}\bar{d}$	$add\bar{d}d\bar{d}\bar{d}$	$dddddd\bar{d}$	$cb\bar{c}bc\bar{c}c$	$cc\bar{c}\bar{c}ccc$	$c\bar{c}\bar{c}\bar{c}cc\bar{c}$
$(2,6,6,42)^*$	$ad\bar{d}d\bar{d}d\bar{d}$	$ad\bar{d}d\bar{d}d\bar{d}$	$cdddd\bar{d}\bar{d}$	$bdddd\bar{d}\bar{d}$	$c\bar{c}dd\bar{c}dd$	$b\bar{b}dd\bar{b}dd$	$ccd\bar{d}\bar{d}dd$	$bbd\bar{d}\bar{d}dd$
$(2,6,12,36)^*$	$\bar{d}c\bar{c}dda\bar{d}$	$\bar{d}c\bar{c}dda\bar{d}$	$db\bar{c}\bar{c}\bar{d}d\bar{d}$	$db\bar{c}\bar{c}\bar{d}d\bar{d}$	$dc\bar{d}b\bar{d}\bar{d}\bar{d}$	$dc\bar{d}b\bar{d}\bar{d}\bar{d}$	$cbdd\bar{d}dd$	$cbdd\bar{d}dd$
$(2,6,22,26)^*$	$\bar{c}b\bar{b}ccc\bar{c}$	$dd\bar{d}dd\bar{d}d$	$cc\bar{b}\bar{b}\bar{c}c\bar{c}$	$\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}dd$	$\bar{c}\bar{c}cb\bar{c}c\bar{c}$	$d\bar{d}\bar{d}ddd\bar{a}$	$bccc\bar{c}cc$	$d\bar{d}dadd\bar{d}$
$(2,7,19,28)^*$	$dddd\bar{d}d\bar{d}$	$accc\bar{c}b\bar{c}$	$\bar{d}ddd\bar{d}d\bar{d}$	$\bar{a}cbc\bar{c}c\bar{c}$	$d\bar{d}dd\bar{d}dd$	$ccbbc\bar{c}\bar{c}$	$ddd\bar{d}\bar{d}dd$	$\bar{c}b\bar{c}\bar{c}\bar{b}b\bar{c}$
$(2,7,21,26)^*$	$bc\bar{c}c\bar{c}c\bar{c}$	$dddd\bar{d}\bar{d}\bar{d}$	$bcccc\bar{c}\bar{c}$	$\bar{d}\bar{d}d\bar{d}\bar{d}\bar{d}d$	$b\bar{b}cc\bar{b}cc$	$d\bar{d}\bar{d}ddd\bar{a}$	$bbc\bar{c}\bar{c}cc$	$add\bar{d}d\bar{d}\bar{d}$
$(2,8,14,32)^*$	$\bar{b}ddbdbb$	$\bar{d}ccdcdd$	$bdd\bar{b}d\bar{b}\bar{b}$	$ccc\bar{c}c\bar{c}\bar{c}$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{c}\bar{d}\bar{d}c\bar{d}cc$	$add\bar{d}d\bar{d}\bar{d}$	$add\bar{d}d\bar{d}\bar{d}$
$(2,9,9,36)^*$	$\bar{b}ddbdbb$	$cdd\bar{d}d\bar{d}\bar{d}$	$bdd\bar{b}d\bar{b}\bar{b}$	$cdd\bar{c}d\bar{c}\bar{c}$	$add\bar{d}d\bar{d}\bar{d}$	$add\bar{d}d\bar{d}\bar{d}$	$bdd\bar{d}d\bar{d}\bar{d}$	$\bar{c}ddcdcc$
$(2,9,17,28)^*$	$\bar{d}d\bar{d}ddd\bar{d}$	$\bar{c}ac\bar{c}c\bar{c}\bar{c}$	$dd\bar{d}\bar{d}\bar{d}d\bar{d}$	$c\bar{c}\bar{b}c\bar{c}\bar{b}\bar{b}$	$dd\bar{d}d\bar{d}\bar{d}\bar{d}$	$\bar{c}bc\bar{b}\bar{c}\bar{c}\bar{c}$	$ddddd\bar{d}d$	$\bar{a}b\bar{c}b\bar{b}\bar{c}b$

Table 3: Continued

type	A_1	A_2	A_3	A_4	A_5	A_6	A_7	A_8
$(2,9,19,26)^*$	$bccc\bar{c}b\bar{c}$	$d\bar{d}\bar{d}dddd$	$\bar{b}cbc\bar{c}c\bar{c}$	$d\bar{d}ddd\bar{d}d$	$b\bar{b}cc\bar{b}cc$	$\bar{d}dd\bar{d}\bar{d}\bar{d}a$	$bbc\bar{c}\bar{c}cc$	$\bar{a}\bar{d}\bar{d}d\bar{d}d\bar{d}$
$(2,13,13,28)^*$	$\bar{b}ddbdbb$	$dcc\bar{c}c\bar{c}\bar{c}$	$\bar{d}\bar{b}\bar{b}d\bar{b}dd$	$\bar{d}\bar{c}\bar{c}d\bar{c}dd$	$dbb\bar{b}b\bar{b}\bar{b}$	$\bar{c}ddcdcc$	$add\bar{d}d\bar{d}\bar{d}$	$add\bar{d}d\bar{d}\bar{d}$
$(2,14,14,26)^*$	$cdd\bar{d}d\bar{d}\bar{d}$	$abb\bar{b}b\bar{b}\bar{b}$	$ddd\bar{d}d\bar{d}\bar{d}$	$b\bar{c}\bar{c}\bar{b}\bar{c}bb$	$bdd\bar{d}d\bar{d}\bar{d}$	$\bar{a}\bar{c}\bar{c}c\bar{c}cc$	$\bar{d}dddddd$	$\bar{c}\bar{b}bc\bar{b}cc$
$(2,14,18,22)$	$\bar{d}ccdcdd$	$\bar{b}\bar{d}\bar{d}b\bar{d}bb$	$dcc\bar{d}c\bar{d}\bar{d}$	$bbb\bar{b}b\bar{b}\bar{b}$	$acc\bar{c}c\bar{c}\bar{c}$	$acc\bar{c}c\bar{c}\bar{c}$	$\bar{d}bbdbdd$	$ddd\bar{d}d\bar{d}\bar{d}$
$(3,6,19,28)^*$	$\bar{d}d\bar{d}ddd\bar{d}$	$c\bar{c}ca\bar{b}\bar{b}\bar{c}$	$d\bar{d}\bar{d}\bar{d}\bar{d}d\bar{d}$	$\bar{a}ccc\bar{c}bc$	$d\bar{d}dd\bar{d}\bar{d}\bar{d}$	$cbacc\bar{c}c$	$ddd\bar{d}ddd$	$c\bar{b}b\bar{c}\bar{c}\bar{c}c$
$(3,7,18,28)^*$	$\bar{c}b\bar{b}ccb\bar{c}$	$d\bar{d}dd\bar{d}dd$	$a\bar{b}\bar{b}\bar{c}c\bar{c}c$	$d\bar{d}\bar{d}ddd\bar{d}$	$cb\bar{c}a\bar{c}\bar{c}\bar{c}$	$\bar{d}\bar{d}ddddd$	$bacc\bar{c}cc$	$d\bar{d}dd\bar{d}dd$
$(3,7,21,25)^*$	$d\bar{d}dd\bar{d}d\bar{d}$	$ccc\bar{c}\bar{b}\bar{c}c$	$a\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$c\bar{c}\bar{c}\bar{c}b\bar{c}\bar{c}$	$a\bar{d}\bar{d}dddd$	$\bar{c}b\bar{c}\bar{c}\bar{c}\bar{c}c$	$\bar{d}dd\bar{d}dda$	$\bar{b}bbcbcc$
$(3,8,14,31)$	$\bar{b}ddbdbb$	$add\bar{d}d\bar{d}\bar{d}$	$bdd\bar{b}d\bar{b}\bar{b}$	$add\bar{d}d\bar{d}\bar{d}$	$\bar{c}ddcdcc$	$\bar{d}\bar{c}\bar{c}d\bar{c}dd$	$acc\bar{c}c\bar{c}\bar{c}$	$cdd\bar{d}d\bar{d}\bar{d}$
$(3,11,17,25)^*$	$a\bar{d}ddddd$	$cb\bar{c}\bar{c}\bar{b}b\bar{c}$	$ad\bar{d}\bar{d}d\bar{d}\bar{d}$	$c\bar{c}c\bar{b}ccb$	$a\bar{d}\bar{d}\bar{d}\bar{d}dd$	$\bar{c}bcc\bar{c}\bar{b}b$	$d\bar{d}d\bar{d}\bar{d}\bar{d}d$	$\bar{b}bb\bar{c}ccc$
$(4,6,18,28)^*$	$\bar{c}b\bar{b}cca\bar{c}$	$\bar{d}\bar{d}d\bar{d}\bar{d}d\bar{d}$	$ca\bar{b}\bar{b}\bar{c}c\bar{c}$	$d\bar{d}dd\bar{d}dd$	$\bar{c}\bar{c}\bar{c}cb\bar{c}a$	$d\bar{d}dddd\bar{d}$	$bacc\bar{c}cc$	$\bar{d}\bar{d}dddd\bar{d}$
$(4,9,19,24)^*$	$a\bar{d}\bar{d}d\bar{d}d\bar{d}$	$\bar{c}\bar{c}ccccc$	$a\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}$	$bc\bar{c}cc\bar{b}\bar{c}$	$a\bar{d}\bar{d}dddd$	$\bar{c}b\bar{b}b\bar{c}\bar{b}\bar{c}$	$\bar{d}dd\bar{d}dda$	$\bar{b}\bar{b}\bar{b}c\bar{c}c\bar{c}$
$(4,10,16,26)$	$da\bar{d}dcdc$	$\bar{b}dd\bar{d}c\bar{b}\bar{c}$	$da\bar{d}d\bar{c}d\bar{c}$	$b\bar{d}\bar{d}dcb\bar{c}$	$da\bar{d}\bar{d}c\bar{d}\bar{c}$	$da\bar{d}\bar{d}\bar{c}dc$	$c\bar{d}c\bar{d}\bar{b}\bar{b}b$	$cdcdbb\bar{b}$
$(4,13,13,26)^*$	$\bar{b}ddbdbb$	$acc\bar{c}c\bar{c}\bar{c}$	$\bar{d}\bar{b}\bar{b}d\bar{b}dd$	$\bar{a}\bar{d}\bar{d}d\bar{d}dd$	$abb\bar{b}b\bar{b}\bar{b}$	$\bar{c}ddcdcc$	$add\bar{d}d\bar{d}\bar{d}$	$dcc\bar{d}c\bar{d}\bar{d}$
$(5,5,23,23)^*$	$ac\bar{c}cc\bar{c}c$	$b\bar{d}\bar{d}dd\bar{d}d$	$a\bar{c}acc\bar{c}\bar{c}$	$\bar{b}d\bar{b}\bar{d}\bar{d}dd$	$a\bar{c}\bar{a}\bar{c}\bar{c}c\bar{c}$	$ddddd\bar{d}\bar{d}$	$\bar{c}\bar{c}ccccc$	$d\bar{d}b\bar{d}\bar{b}\bar{d}\bar{d}$
$(5,9,14,28)^*$	$a\bar{b}b\bar{c}ccc$	$\bar{d}d\bar{d}\bar{d}dd\bar{d}$	$ab\bar{c}ac\bar{b}\bar{c}$	$\bar{d}\bar{d}\bar{d}\bar{d}\bar{d}dd$	$b\bar{b}\bar{b}\bar{b}\bar{c}\bar{b}c$	$\bar{d}d\bar{d}d\bar{d}\bar{d}\bar{d}$	$\bar{a}ccac\bar{c}c$	$d\bar{d}ddd\bar{d}d$
$(5,9,19,23)^*$	$\bar{c}b\bar{b}ccc\bar{c}$	$ddd\bar{d}\bar{d}dd$	$\bar{c}c\bar{c}cb\bar{b}\bar{b}$	$\bar{a}\bar{a}ad\bar{d}\bar{d}\bar{d}$	$cb\bar{c}b\bar{c}\bar{c}\bar{c}$	$\bar{d}dddd\bar{d}d$	$bbcc\bar{c}cc$	$\bar{d}a\bar{d}\bar{a}d\bar{d}d$
$(6,6,22,22)^*$	$cc\bar{c}c\bar{c}c\bar{c}$	$d\bar{d}d\bar{d}d\bar{d}\bar{d}$	$acccc\bar{c}\bar{c}$	$\bar{b}\bar{d}\bar{d}\bar{d}ddd$	$a\bar{a}cc\bar{a}cc$	$\bar{b}b\bar{d}\bar{d}b\bar{d}\bar{d}$	$aac\bar{c}\bar{c}cc$	$\bar{b}\bar{b}\bar{d}dd\bar{d}\bar{d}$
$(6,7,7,36)^*$	$\bar{d}b\bar{b}ddb\bar{d}$	$\bar{d}c\bar{c}ddc\bar{d}$	$da\bar{b}\bar{b}\bar{d}d\bar{d}$	$da\bar{c}\bar{c}d\bar{d}d$	$db\bar{d}a\bar{d}\bar{d}d$	$\bar{d}\bar{d}\bar{c}a\bar{d}\bar{d}d$	$badd\bar{d}dd$	$d\bar{a}ddd\bar{d}\bar{c}$
$(6,12,16,22)^*$	$cb\bar{c}cbcb$	$adddd\bar{d}\bar{d}$	$cb\bar{c}c\bar{b}c\bar{b}$	$daa\bar{d}\bar{d}dd$	$\bar{c}bcb\bar{c}\bar{c}b$	$d\bar{d}d\bar{d}\bar{d}d\bar{d}$	$cb\bar{c}\bar{c}\bar{b}cb$	$\bar{d}\bar{a}a\bar{d}\bar{d}a\bar{d}$
$(7,7,16,26)^*$	$\bar{d}c\bar{a}aac\bar{d}$	$\bar{d}cbdb\bar{c}d$	$\bar{a}dc\bar{d}dcd$	$b\bar{d}\bar{c}d\bar{d}\bar{c}\bar{d}$	$a\bar{d}\bar{c}ddcd$	$\bar{b}dc\bar{d}\bar{d}\bar{c}\bar{d}$	$\bar{d}cada\bar{c}d$	$\bar{d}c\bar{b}bbc\bar{d}$
$(7,8,20,21)^*$	$cc\bar{c}cbcb$	$dd\bar{d}\bar{a}\bar{d}dd$	$cc\bar{c}c\bar{b}c\bar{b}$	$dd\bar{d}\bar{d}a\bar{d}\bar{d}$	$cc\bar{c}\bar{c}b\bar{c}\bar{b}$	$dd\bar{d}\bar{d}ad\bar{d}$	$c\bar{c}\bar{c}\bar{b}\bar{c}bc$	$\bar{a}aadadd$
$(9,9,10,28)^*$	$d\bar{d}\bar{d}ddd\bar{d}$	$caaa\bar{c}\bar{a}a$	$\bar{d}d\bar{d}dddd$	$\bar{c}cc\bar{b}\bar{a}ba$	$d\bar{d}ddd\bar{d}d$	$bbb\bar{a}a\bar{c}c$	$\bar{d}ddddd\bar{d}$	$\bar{c}\bar{c}b\bar{b}b\bar{c}\bar{b}$
$(9,9,19,19)^*$	$\bar{a}ccacaa$	$bdd\bar{d}d\bar{d}\bar{d}$	$acc\bar{a}c\bar{a}\bar{a}$	$bdd\bar{b}d\bar{b}\bar{b}$	$acc\bar{c}c\bar{c}\bar{c}$	$\bar{b}ddbdbb$	$ccc\bar{c}c\bar{c}\bar{c}$	$\bar{d}\bar{d}\bar{d}d\bar{d}dd$
$(9,14,14,19)^*$	$\bar{a}ddadaa$	$\bar{b}\bar{c}\bar{c}b\bar{c}bb$	$add\bar{a}d\bar{a}\bar{a}$	$\bar{b}\bar{c}\bar{c}c\bar{c}cc$	$add\bar{d}d\bar{d}\bar{d}$	$c\bar{b}\bar{b}\bar{c}\bar{b}\bar{c}\bar{c}$	$ddd\bar{d}d\bar{d}\bar{d}$	$\bar{c}\bar{b}\bar{b}b\bar{b}bb$
$(11,11,17,17)^*$	$\bar{a}c\bar{c}ca\bar{a}c$	$b\bar{d}ddd\bar{b}d$	$cccac\bar{c}\bar{c}$	$b\bar{d}bbbbd$	$a\bar{c}ccc\bar{a}c$	$\bar{b}\bar{d}ddb\bar{b}d$	$\bar{a}aaaca\bar{c}$	$dbd\bar{d}\bar{d}dd$
$(12,12,16,16)^*$	$ca\bar{c}caca$	$\bar{b}db\bar{d}\bar{d}b\bar{d}$	$ca\bar{c}c\bar{a}c\bar{a}$	$db\bar{d}d\bar{b}d\bar{b}$	$\bar{a}ca\bar{c}\bar{c}a\bar{c}$	$db\bar{d}dbdb$	$ca\bar{c}\bar{c}\bar{a}\bar{c}a$	$db\bar{d}\bar{d}\bar{b}\bar{d}b$
$(13,13,15,15)^*$	$\bar{a}ccacaa$	$dbb\bar{b}b\bar{b}\bar{b}$	$\bar{c}\bar{a}\bar{a}c\bar{a}cc$	$\bar{d}\bar{b}\bar{b}d\bar{b}dd$	$caa\bar{a}a\bar{a}\bar{a}$	$\bar{b}ddbdbb$	$ccc\bar{c}c\bar{c}\bar{c}$	$\bar{d}\bar{d}\bar{d}d\bar{d}dd$
$(13,14,14,15)^*$	$\bar{c}bbcbcc$	$daa\bar{a}a\bar{a}\bar{a}$	$\bar{b}\bar{c}\bar{c}b\bar{c}bb$	$a\bar{d}\bar{d}\bar{a}\bar{d}\bar{a}\bar{a}$	$bcc\bar{c}c\bar{c}\bar{c}$	$daa\bar{d}a\bar{d}\bar{d}$	$cbb\bar{b}b\bar{b}\bar{b}$	$ddd\bar{d}d\bar{d}\bar{d}$
$(2,11,43)$	$\bar{b}ccbcbb$	$bcc\bar{b}c\bar{b}\bar{b}$	$acc\bar{c}c\bar{c}\bar{c}$	$acc\bar{c}c\bar{c}\bar{c}$	$b\bar{c}ccccc$	$c\bar{c}c\bar{c}c\bar{c}\bar{c}$	$b\bar{c}\bar{c}\bar{c}\bar{c}cc$	$bc\bar{c}\bar{c}c\bar{c}\bar{c}$
$(4,5,47)$	$cb\bar{c}cacc$	$cb\bar{c}\bar{c}a\bar{c}\bar{c}$	$cb\bar{c}c\bar{a}c\bar{c}$	$\bar{c}cc\bar{c}\bar{c}\bar{c}\bar{c}$	$cb\bar{c}\bar{c}\bar{a}cc$	$c\bar{c}cc\bar{c}cc$	$\bar{c}ccccc\bar{c}$	$bc\bar{c}c\bar{c}c\bar{c}$
$(4,23,29)$	$bb\bar{b}babc$	$bb\bar{b}b\bar{a}b\bar{c}$	$bb\bar{b}\bar{b}\bar{a}bc$	$bb\bar{b}\bar{b}ab\bar{c}$	$c\bar{c}c\bar{c}c\bar{c}\bar{c}$	$b\bar{c}ccccc$	$bc\bar{c}\bar{c}c\bar{c}\bar{c}$	$b\bar{c}\bar{c}\bar{c}ccc$
$(5,6,45)$	$bc\bar{c}c\bar{c}c\bar{c}$	$c\bar{c}cccc\bar{c}$	$cb\bar{b}\bar{b}a\bar{c}a$	$cc\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}$	$c\bar{c}\bar{c}cccc$	$\bar{a}\bar{c}\bar{c}c\bar{c}a\bar{c}$	$ab\bar{c}bc\bar{c}c$	$ccc\bar{c}\bar{c}cc$
$(5,8,43)$	$\bar{b}ccbcbb$	$cc\bar{c}c\bar{c}\bar{c}c$	$bcc\bar{b}c\bar{b}\bar{b}$	$cc\bar{c}c\bar{c}\bar{c}c$	$\bar{c}\bar{c}ccccc$	$ca\bar{c}ac\bar{c}c$	$c\bar{c}cca\bar{a}a$	$\bar{c}\bar{c}\bar{c}c\bar{c}c\bar{c}$
$(6,9,41)^*$	$\bar{c}b\bar{b}cca\bar{c}$	$ccc\bar{c}\bar{c}cc$	$cb\bar{b}\bar{b}\bar{c}c\bar{c}$	$ca\bar{a}\bar{a}c\bar{c}c$	$cb\bar{c}b\bar{c}\bar{c}\bar{c}$	$\bar{c}\bar{a}c\bar{a}\bar{c}c\bar{c}$	$bbcc\bar{c}cc$	$\bar{c}c\bar{c}\bar{c}c\bar{c}c$
$(6,20,30)^*$	$a\bar{b}bbcbc$	$a\bar{b}bbcbc$	$ab\bar{b}\bar{b}c\bar{c}\bar{c}$	$ab\bar{b}\bar{b}c\bar{c}\bar{c}$	$a\bar{c}\bar{b}\bar{c}\bar{c}cc$	$a\bar{c}\bar{b}\bar{c}\bar{c}cc$	$b\bar{c}b\bar{c}c\bar{c}\bar{c}$	$b\bar{c}b\bar{c}c\bar{c}\bar{c}$
$(8,11,37)$	$\bar{a}ccacaa$	$bcc\bar{c}c\bar{c}\bar{c}$	$acc\bar{a}c\bar{a}\bar{a}$	$bcc\bar{c}c\bar{c}\bar{c}$	$ccc\bar{c}c\bar{c}\bar{c}$	$bcc\bar{c}c\bar{c}\bar{c}$	$bcc\bar{b}c\bar{b}\bar{b}$	$\bar{b}ccbcbb$
$(10,13,33)$	$bcbcaa\bar{a}$	$b\bar{c}b\bar{c}\bar{a}\bar{a}a$	$b\bar{a}b\bar{a}cc\bar{c}$	$ba\bar{b}a\bar{c}\bar{c}c$	$c\bar{c}ccc\bar{c}c$	$bb\bar{b}\bar{c}ccc$	$c\bar{c}\bar{c}cccc$	$\bar{b}c\bar{b}c\bar{c}c\bar{c}$
$(10,15,31)$	$cbcbaa\bar{a}$	$c\bar{b}c\bar{b}\bar{a}\bar{a}a$	$\bar{c}\bar{a}\bar{c}abb\bar{b}$	$ca\bar{c}a\bar{b}\bar{b}b$	$c\bar{c}ccc\bar{c}c$	$bb\bar{b}\bar{c}ccc$	$c\bar{c}\bar{c}cccc$	$\bar{b}c\bar{b}c\bar{c}c\bar{c}$
$(13,21,22)$	$\bar{c}bbcbcc$	$baa\bar{a}a\bar{a}\bar{a}$	$cbb\bar{b}b\bar{b}\bar{b}$	$cbb\bar{c}b\bar{c}\bar{c}$	$bbb\bar{b}b\bar{b}\bar{b}$	$\bar{c}aacacc$	$bcc\bar{c}c\bar{c}\bar{c}$	$\bar{a}\bar{c}\bar{c}a\bar{c}aa$
$(15,18,23)^*$	$\bar{c}bbcbcc$	$caa\bar{a}a\bar{a}\bar{a}$	$cbb\bar{c}b\bar{c}\bar{c}$	$\bar{c}aacacc$	$abb\bar{b}b\bar{b}\bar{b}$	$acc\bar{c}c\bar{c}\bar{c}$	$cbb\bar{b}b\bar{b}\bar{b}$	$\bar{a}\bar{c}\bar{c}a\bar{c}aa$

Table 4: Complementary negacyclic matrices for given OD of order 56 in 3 variables

type	A_1	A_2	A_3	A_4
$(1,9,46)$	$\bar{c}\bar{c}\bar{c}cc\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}cc\bar{c}\bar{c}$	$\bar{b}c\bar{b}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}c\bar{c}\bar{c}$	$\bar{b}\bar{c}c\bar{c}\bar{c}c\bar{c}\bar{c}c\bar{c}b\bar{c}\bar{b}\bar{c}$	$\bar{a}\bar{c}\bar{b}\bar{c}c\bar{c}ccc\bar{c}c\bar{c}\bar{b}\bar{c}$
$(1,23,32)$	$\bar{a}\bar{c}\bar{c}cc\bar{c}c\bar{c}c\bar{c}cc\bar{c}\bar{c}$	$\bar{b}\bar{b}c\bar{b}bcbcc\bar{b}b\bar{c}\bar{c}\bar{b}$	$\bar{b}\bar{b}ccc\bar{b}b\bar{b}\bar{b}bccc\bar{b}$	$\bar{b}\bar{b}\bar{b}\bar{b}\bar{c}c\bar{c}\bar{c}\bar{b}\bar{b}\bar{c}c\bar{b}\bar{c}$
$(1,24,31)$	$\bar{a}\bar{c}c\bar{c}\bar{c}\bar{c}c\bar{b}c\bar{c}\bar{c}\bar{c}c\bar{c}$	$\bar{b}\bar{b}\bar{c}bcb\bar{b}\bar{c}\bar{c}b\bar{b}\bar{c}cb$	$\bar{b}\bar{b}cccc\bar{b}b\bar{b}\bar{b}ccb\bar{b}$	$\bar{b}\bar{b}\bar{b}\bar{b}\bar{c}c\bar{c}\bar{c}\bar{b}\bar{b}c\bar{c}\bar{b}c$
$(2,5,49)$	$\bar{a}c\bar{b}\bar{c}cc\bar{c}c\bar{c}cc\bar{c}\bar{c}c$	$\bar{a}\bar{c}\bar{c}cccc\bar{c}cccc\bar{c}\bar{b}\bar{c}$	$\bar{b}c\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}cccccc$	$\bar{b}c\bar{c}c\bar{c}cc\bar{c}c\bar{c}\bar{c}c\bar{c}\bar{c}$
$(2,7,47)$	$\bar{b}c\bar{b}c\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}c$	$\bar{b}cc\bar{c}\bar{b}c\bar{c}cc\bar{c}\bar{c}\bar{c}c\bar{c}$	$\bar{a}c\bar{b}\bar{c}c\bar{c}\bar{c}ccc\bar{c}\bar{c}c\bar{c}$	$\bar{a}\bar{c}\bar{c}c\bar{c}ccc\bar{c}\bar{c}c\bar{c}\bar{b}\bar{c}$
$(2,11,43)$	$\bar{b}cc\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}bc\bar{b}\bar{c}\bar{c}$	$\bar{b}c\bar{b}\bar{c}ccc\bar{c}\bar{c}\bar{b}\bar{c}c\bar{c}c$	$\bar{a}c\bar{c}\bar{b}c\bar{c}c\bar{c}\bar{c}\bar{b}c\bar{c}\bar{c}\bar{c}$	$\bar{a}\bar{c}\bar{c}\bar{c}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}c\bar{b}\bar{c}c$
$(2,23,31)$	$\bar{a}\bar{b}\bar{c}\bar{c}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}cb$	$\bar{b}\bar{b}ccc\bar{b}\bar{c}b\bar{b}b\bar{c}c\bar{b}\bar{c}$	$\bar{b}\bar{b}c\bar{b}c\bar{c}\bar{b}bbc\bar{b}c\bar{c}\bar{c}$	$\bar{a}bc\bar{b}\bar{c}bcbcc\bar{c}\bar{b}\bar{c}\bar{b}$
$(3,4,49)$	$\bar{b}c\bar{c}c\bar{c}\bar{b}c\bar{c}\bar{c}\bar{c}c\bar{c}\bar{c}c$	$\bar{b}c\bar{c}\bar{c}\bar{c}ccc\bar{c}\bar{b}\bar{c}\bar{c}\bar{c}c$	$\bar{a}ccc\bar{c}\bar{c}\bar{c}ac\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}$	$\bar{a}c\bar{c}c\bar{c}\bar{c}cc\bar{c}\bar{c}\bar{c}\bar{c}cc$
$(3,6,47)$	$\bar{a}cb\bar{c}\bar{b}cccccccc\bar{c}$	$\bar{a}c\bar{c}cc\bar{c}\bar{c}\bar{c}\bar{c}c\bar{b}\bar{c}\bar{c}c$	$\bar{b}c\bar{b}c\bar{c}cc\bar{c}\bar{c}\bar{c}c\bar{c}cc$	$\bar{a}\bar{c}\bar{b}cc\bar{c}c\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}c$
$(3,7,46)$	$\bar{a}c\bar{c}\bar{c}cc\bar{c}a\bar{b}cccc\bar{c}$	$\bar{a}c\bar{c}\bar{c}\bar{c}c\bar{b}c\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}b$	$\bar{b}\bar{c}c\bar{c}cc\bar{c}c\bar{c}\bar{c}\bar{c}c\bar{b}\bar{c}$	$\bar{b}\bar{c}bcc\bar{c}c\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}$
$(3,10,43)$	$\bar{b}\bar{b}ccccc\bar{b}\bar{c}\bar{c}ccc\bar{b}$	$\bar{a}c\bar{c}bc\bar{c}\bar{c}\bar{c}c\bar{b}c\bar{c}cc$	$\bar{a}\bar{c}c\bar{b}\bar{c}c\bar{c}\bar{c}\bar{c}c\bar{c}\bar{b}\bar{c}\bar{c}$	$\bar{a}c\bar{c}\bar{b}\bar{c}\bar{b}c\bar{c}\bar{c}\bar{c}c\bar{c}\bar{c}c$
$(3,11,42)$	$\bar{b}c\bar{b}cccc\bar{c}cb\bar{c}\bar{c}\bar{c}\bar{b}$	$\bar{a}ccb\bar{b}\bar{c}\bar{c}\bar{b}\bar{c}\bar{c}cccc$	$\bar{a}ccb\bar{c}cc\bar{c}\bar{b}c\bar{c}\bar{c}\bar{c}c$	$\bar{a}\bar{c}\bar{c}cb\bar{c}\bar{b}cc\bar{c}c\bar{c}cc$
$(3,21,32)$	$\bar{b}ccccc\bar{c}\bar{c}\bar{c}bc\bar{b}\bar{c}\bar{c}$	$\bar{a}b\bar{c}cb\bar{b}bc\bar{c}\bar{c}c\bar{b}\bar{b}\bar{c}$	$\bar{a}ccb\bar{b}\bar{b}\bar{b}\bar{c}c\bar{b}bc\bar{c}c$	$\bar{a}\bar{c}\bar{c}\bar{b}b\bar{c}\bar{c}\bar{c}c\bar{b}\bar{c}bbb$
$(3,22,31)$	$\bar{a}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}cc$	$\bar{a}cb\bar{b}b\bar{b}bcccb\bar{c}\bar{c}\bar{b}$	$\bar{a}\bar{b}\bar{b}\bar{c}c\bar{c}c\bar{b}\bar{b}\bar{b}c\bar{c}\bar{b}c$	$\bar{b}\bar{b}\bar{c}c\bar{b}cb\bar{b}c\bar{c}bc\bar{c}\bar{b}$
$(3,24,29)$	$\bar{a}\bar{b}b\bar{c}c\bar{b}\bar{b}\bar{c}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}$	$\bar{b}\bar{c}bcc\bar{b}\bar{c}c\bar{c}\bar{c}b\bar{c}\bar{c}b$	$\bar{a}\bar{c}c\bar{b}\bar{b}\bar{c}\bar{b}b\bar{c}ccbb\bar{b}$	$\bar{a}\bar{b}\bar{c}bbcbc\bar{b}\bar{c}c\bar{b}c\bar{b}$
$(4,5,47)$	$\bar{a}ccc\bar{c}\bar{c}\bar{c}a\bar{b}\bar{c}\bar{c}\bar{c}c\bar{c}$	$\bar{b}cc\bar{c}\bar{c}c\bar{c}\bar{b}c\bar{c}\bar{c}\bar{c}c\bar{c}$	$\bar{a}\bar{c}\bar{c}c\bar{c}cccc\bar{c}cc\bar{c}\bar{b}$	$\bar{a}\bar{c}cc\bar{c}c\bar{b}\bar{c}\bar{c}cc\bar{c}\bar{c}\bar{c}$
$(4,11,41)$	$\bar{a}cc\bar{b}\bar{c}\bar{b}\bar{c}c\bar{c}\bar{c}\bar{c}c\bar{c}\bar{b}$	$\bar{a}ccb\bar{c}\bar{c}ccc\bar{b}c\bar{b}cc$	$\bar{a}\bar{c}\bar{c}\bar{b}cb\bar{c}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}cc$	$\bar{a}\bar{b}c\bar{c}c\bar{c}c\bar{c}cb\bar{c}cc\bar{c}$
$(4,15,37)$	$\bar{a}\bar{c}\bar{b}ccb\bar{b}\bar{b}cc\bar{c}cc\bar{c}$	$\bar{a}bccc\bar{b}\bar{c}\bar{c}\bar{c}cbc\bar{b}\bar{c}$	$\bar{a}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}c\bar{c}\bar{c}\bar{c}\bar{b}b\bar{c}b$	$\bar{a}c\bar{c}c\bar{c}c\bar{c}c\bar{b}\bar{b}c\bar{b}\bar{c}c$
$(4,23,29)$	$\bar{b}\bar{c}\bar{b}c\bar{c}\bar{b}\bar{c}bccc\bar{b}c\bar{b}$	$\bar{a}\bar{b}\bar{c}c\bar{c}\bar{b}\bar{c}\bar{a}cb\bar{c}b\bar{c}\bar{c}$	$\bar{a}cbb\bar{b}cb\bar{a}\bar{c}\bar{b}c\bar{b}cc$	$\bar{b}\bar{b}\bar{b}\bar{c}\bar{c}c\bar{b}\bar{c}b\bar{b}\bar{c}c\bar{b}c$
$(5,6,45)$	$\bar{a}\bar{c}\bar{b}c\bar{c}\bar{c}c\bar{c}c\bar{c}\bar{c}\bar{a}c\bar{c}$	$\bar{a}\bar{c}b\bar{c}cccc\bar{c}ccc\bar{b}\bar{c}$	$\bar{a}cc\bar{a}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}cc$	$\bar{b}\bar{c}cc\bar{c}\bar{c}c\bar{c}c\bar{c}\bar{b}\bar{c}c\bar{c}$
$(5,7,44)$	$\bar{a}\bar{b}cc\bar{c}\bar{c}\bar{c}c\bar{b}c\bar{c}cc\bar{c}$	$\bar{a}\bar{c}c\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}\bar{c}cc\bar{b}\bar{c}c$	$\bar{a}c\bar{c}\bar{b}c\bar{c}cbc\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}$	$\bar{a}c\bar{c}\bar{c}cc\bar{c}\bar{a}\bar{c}\bar{c}ccc\bar{b}$
$(5,8,43)$	$\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}c\bar{a}ccccc\bar{b}$	$\bar{a}\bar{b}\bar{c}cc\bar{c}\bar{c}cc\bar{c}c\bar{c}\bar{c}b$	$\bar{a}cccc\bar{c}cbc\bar{b}\bar{c}\bar{c}\bar{c}c$	$\bar{a}b\bar{c}c\bar{c}cc\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}\bar{b}$
$(5,9,42)$	$\bar{a}b\bar{c}\bar{c}c\bar{b}cc\bar{b}\bar{c}cc\bar{c}\bar{c}$	$\bar{a}ccccb\bar{b}ccbcccc$	$\bar{a}\bar{c}bc\bar{c}\bar{c}c\bar{a}c\bar{c}\bar{c}\bar{c}\bar{c}c$	$\bar{a}\bar{b}\bar{b}\bar{c}cc\bar{c}c\bar{c}c\bar{c}\bar{c}\bar{c}c$
$(5,11,40)$	$\bar{a}ccc\bar{b}\bar{c}c\bar{b}c\bar{c}\bar{c}\bar{b}\bar{c}c$	$\bar{a}\bar{c}\bar{c}\bar{c}b\bar{c}c\bar{b}c\bar{c}\bar{b}cc\bar{c}$	$\bar{a}\bar{c}\bar{c}c\bar{b}\bar{c}cb\bar{c}\bar{c}\bar{c}\bar{b}\bar{c}c$	$\bar{a}c\bar{c}\bar{c}\bar{b}c\bar{c}\bar{a}c\bar{c}\bar{b}\bar{c}\bar{c}\bar{c}$
$(5,13,38)$	$\bar{a}b\bar{b}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}c$	$\bar{a}bc\bar{c}\bar{c}\bar{c}\bar{b}\bar{a}cccccb$	$\bar{a}c\bar{b}\bar{c}cc\bar{b}c\bar{c}\bar{c}\bar{c}c\bar{b}c$	$\bar{a}\bar{c}\bar{c}cbbc\bar{c}\bar{c}bc\bar{c}\bar{c}c$
$(5,14,37)$	$\bar{a}ccc\bar{c}c\bar{c}b\bar{b}\bar{b}\bar{c}\bar{c}bc$	$\bar{a}c\bar{b}b\bar{b}\bar{b}cc\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}c$	$\bar{a}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}\bar{a}cb\bar{c}bc\bar{c}$	$\bar{a}bbc\bar{c}bc\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}c$
$(5,16,35)$	$\bar{a}b\bar{b}cc\bar{b}\bar{c}\bar{a}\bar{c}\bar{c}\bar{c}\bar{c}b\bar{c}$	$\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}c\bar{b}\bar{b}c\bar{c}\bar{b}\bar{c}$	$\bar{a}cbcc\bar{c}\bar{b}\bar{b}\bar{c}c\bar{b}c\bar{c}\bar{c}$	$\bar{a}\bar{c}c\bar{c}\bar{c}cc\bar{b}bcbc\bar{b}\bar{c}$
$(5,17,34)$	$\bar{a}cccb\bar{b}\bar{c}\bar{a}\bar{b}cc\bar{c}c\bar{b}$	$\bar{a}\bar{b}b\bar{c}\bar{c}\bar{c}b\bar{b}\bar{c}\bar{b}cccc$	$\bar{a}bbc\bar{b}\bar{b}c\bar{c}\bar{c}\bar{c}\bar{c}cc\bar{c}$	$\bar{a}\bar{c}\bar{b}\bar{b}c\bar{c}\bar{b}\bar{c}c\bar{b}\bar{c}c\bar{c}\bar{c}$
$(5,19,32)$	$\bar{a}cc\bar{c}\bar{c}bcb\bar{b}c\bar{b}\bar{c}c\bar{b}$	$\bar{a}\bar{b}cc\bar{b}\bar{c}c\bar{a}ccbbc\bar{c}$	$\bar{a}\bar{c}\bar{c}bccb\bar{b}\bar{c}\bar{c}cbbc$	$\bar{a}\bar{c}b\bar{c}\bar{c}\bar{c}c\bar{b}bb\bar{c}\bar{b}\bar{c}c$
$(5,20,31)$	$\bar{a}\bar{c}\bar{b}\bar{c}c\bar{a}\bar{c}ccc\bar{b}cb\bar{c}$	$\bar{a}\bar{b}\bar{c}b\bar{c}b\bar{c}\bar{b}\bar{c}bc\bar{b}\bar{c}b$	$\bar{a}cc\bar{c}c\bar{b}cb\bar{c}\bar{a}cbcc$	$\bar{b}\bar{c}\bar{b}\bar{c}bc\bar{b}cbc\bar{b}c\bar{b}\bar{c}$
$(5,21,30)$	$\bar{b}\bar{b}\bar{c}\bar{b}bc\bar{c}cccc\bar{c}\bar{c}c$	$\bar{a}\bar{c}cb\bar{b}\bar{c}\bar{c}\bar{c}\bar{a}b\bar{b}\bar{c}c\bar{c}$	$\bar{a}cccbb\bar{a}\bar{b}\bar{c}b\bar{b}b\bar{c}c$	$\bar{a}\bar{b}\bar{b}b\bar{c}c\bar{b}cb\bar{b}cc\bar{b}\bar{c}$
$(5,22,29)$	$\bar{a}\bar{b}\bar{c}\bar{c}\bar{b}b\bar{b}\bar{b}\bar{b}ccc\bar{c}c$	$\bar{a}\bar{a}cc\bar{b}b\bar{c}\bar{c}c\bar{c}\bar{c}\bar{c}c\bar{b}$	$\bar{b}\bar{b}\bar{c}\bar{b}cbcc\bar{b}\bar{b}b\bar{c}c\bar{c}$	$\bar{a}\bar{c}\bar{b}bb\bar{c}\bar{b}c\bar{b}\bar{c}\bar{b}\bar{c}c\bar{a}$
$(5,24,27)$	$\bar{a}\bar{b}\bar{b}\bar{c}bcccc\bar{c}cb\bar{b}b$	$\bar{a}cc\bar{c}c\bar{c}\bar{c}\bar{c}\bar{b}b\bar{c}\bar{b}bc$	$\bar{a}\bar{c}c\bar{b}\bar{c}\bar{c}bcbc\bar{b}\bar{b}c\bar{b}$	$\bar{a}c\bar{b}\bar{b}cbb\bar{a}b\bar{b}b\bar{c}cb$
$(6,9,41)$	$\bar{a}cc\bar{c}\bar{c}cb\bar{c}\bar{c}c\bar{c}c\bar{b}\bar{b}$	$\bar{a}ccc\bar{b}\bar{c}\bar{c}\bar{c}bcc\bar{c}bc$	$\bar{a}\bar{a}\bar{b}bcccccc\bar{c}ccc$	$\bar{a}c\bar{c}\bar{b}c\bar{c}c\bar{c}c\bar{c}cc\bar{c}\bar{a}$
$(6,11,39)$	$\bar{a}\bar{b}cccc\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}b\bar{a}\bar{b}$	$\bar{a}\bar{c}\bar{b}c\bar{c}\bar{c}\bar{c}cb\bar{c}bc\bar{c}c$	$\bar{a}cbcc\bar{c}cc\bar{b}\bar{c}\bar{b}c\bar{c}c$	$\bar{a}c\bar{a}\bar{b}\bar{c}\bar{c}c\bar{c}c\bar{c}\bar{c}\bar{c}c\bar{b}$
$(6,16,34)$	$\bar{a}\bar{a}\bar{b}\bar{c}b\bar{b}cccc\bar{b}\bar{c}c\bar{c}$	$\bar{a}ccc\bar{b}bbccc\bar{b}\bar{c}\bar{c}\bar{a}$	$\bar{a}c\bar{c}c\bar{c}\bar{b}c\bar{b}\bar{c}cc\bar{b}\bar{c}\bar{b}$	$\bar{a}\bar{c}b\bar{b}\bar{b}\bar{b}\bar{c}cc\bar{c}\bar{c}c\bar{c}c$
$(6,17,33)$	$\bar{a}c\bar{b}\bar{c}\bar{b}\bar{a}cc\bar{c}bc\bar{c}\bar{c}\bar{b}$	$\bar{a}\bar{c}ccccc\bar{c}b\bar{b}bb\bar{c}\bar{b}$	$\bar{a}cb\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}\bar{a}\bar{c}bcc$	$\bar{a}\bar{c}bb\bar{c}c\bar{c}c\bar{c}c\bar{b}\bar{b}c\bar{c}$
$(6,21,29)$	$\bar{a}\bar{c}bc\bar{b}\bar{b}\bar{a}\bar{c}\bar{b}\bar{b}\bar{c}c\bar{c}\bar{c}$	$\bar{a}cc\bar{b}b\bar{c}\bar{c}b\bar{a}\bar{c}bccb$	$\bar{a}\bar{c}\bar{b}b\bar{c}b\bar{b}\bar{c}\bar{c}\bar{c}b\bar{b}\bar{c}\bar{c}$	$\bar{a}c\bar{c}\bar{b}c\bar{c}c\bar{c}cbc\bar{b}\bar{b}\bar{b}$
$(6,23,27)$	$\bar{a}\bar{c}\bar{b}\bar{b}\bar{c}\bar{b}bc\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}$	$\bar{a}\bar{c}\bar{b}\bar{c}\bar{c}\bar{b}\bar{b}cb\bar{b}\bar{c}c\bar{b}\bar{b}$	$\bar{a}cb\bar{a}\bar{b}\bar{c}b\bar{b}c\bar{b}\bar{b}\bar{c}\bar{c}b$	$\bar{a}\bar{b}b\bar{c}b\bar{c}b\bar{c}cc\bar{c}\bar{a}c\bar{b}$
$(7,8,41)$	$\bar{a}bc\bar{c}\bar{c}cc\bar{c}b\bar{c}c\bar{b}\bar{c}\bar{c}$	$\bar{a}c\bar{b}\bar{c}ccccc\bar{b}\bar{c}ccb$	$\bar{a}\bar{c}\bar{a}cc\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{a}c$	$\bar{a}c\bar{c}c\bar{a}\bar{c}c\bar{c}c\bar{c}c\bar{c}\bar{b}c$
$(7,9,40)$	$\bar{a}ccc\bar{c}\bar{b}\bar{c}c\bar{c}\bar{c}\bar{a}c\bar{b}c$	$\bar{a}\bar{c}bc\bar{a}\bar{c}c\bar{c}c\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}$	$\bar{a}c\bar{b}c\bar{c}cc\bar{b}\bar{c}cccbc$	$\bar{a}\bar{c}\bar{c}c\bar{c}bc\bar{a}c\bar{b}\bar{c}ccc$

Table 4: Continued

type	A_1	A_2	A_3	A_4
$(7,10,39)$	$\bar{a}\bar{b}\bar{c}\bar{b}cc\bar{a}\bar{c}\bar{c}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}$	$\bar{a}c\bar{b}bbc\bar{c}\bar{c}\bar{c}c\bar{c}cc\bar{c}$	$\bar{a}\bar{c}\bar{c}c\bar{c}c\bar{c}\bar{a}\bar{c}bc\bar{b}cc$	$\bar{a}b\bar{c}bc\bar{c}cc\bar{a}\bar{c}cc\bar{c}\bar{c}$
$(7,15,34)$	$\bar{a}\bar{b}\bar{c}\bar{b}\bar{a}cb\bar{c}ccc\bar{c}\bar{c}\bar{b}$	$\bar{a}b\bar{a}ccb\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{a}\bar{b}$	$\bar{a}\bar{c}c\bar{c}c\bar{c}\bar{b}c\bar{c}bb\bar{c}\bar{c}b$	$\bar{a}\bar{c}\bar{b}b\bar{c}\bar{b}c\bar{c}c\bar{c}\bar{b}cc\bar{c}$
$(7,16,33)$	$\bar{a}\bar{c}cc\bar{c}\bar{c}b\bar{a}\bar{c}\bar{b}\bar{b}\bar{c}\bar{b}\bar{c}$	$\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}cc\bar{a}\bar{b}c\bar{b}cc\bar{b}$	$\bar{a}cccc\bar{c}\bar{c}\bar{a}\bar{b}cbc\bar{b}\bar{b}$	$\bar{a}c\bar{c}b\bar{b}bc\bar{c}c\bar{c}c\bar{c}bc$
$(7,17,32)$	$\bar{a}c\bar{c}bcc\bar{c}\bar{c}\bar{a}bc\bar{b}\bar{b}c$	$\bar{a}cc\bar{b}bcc\bar{a}c\bar{c}\bar{b}\bar{b}\bar{c}c$	$\bar{a}cb\bar{b}b\bar{c}\bar{c}cbcc\bar{c}b\bar{c}$	$\bar{a}\bar{c}\bar{c}c\bar{c}\bar{c}\bar{a}\bar{c}\bar{b}\bar{b}c\bar{b}cb$
$(7,19,30)$	$\bar{a}\bar{b}\bar{c}cc\bar{a}\bar{c}c\bar{c}cccc\bar{b}$	$\bar{a}\bar{c}ccbcc\bar{a}b\bar{b}\bar{b}b\bar{c}b$	$\bar{a}\bar{c}b\bar{c}\bar{c}cb\bar{c}b\bar{a}c\bar{c}\bar{b}\bar{c}$	$\bar{a}\bar{b}\bar{b}c\bar{b}\bar{c}\bar{b}\bar{c}cbb\bar{b}cc$
$(7,20,29)$	$\bar{a}\bar{c}\bar{b}\bar{c}cc\bar{c}\bar{c}\bar{c}\bar{c}b\bar{b}\bar{c}c$	$\bar{a}cbcb\bar{c}\bar{a}c\bar{b}bbb\bar{c}\bar{c}$	$\bar{a}\bar{b}c\bar{c}b\bar{c}b\bar{a}\bar{c}\bar{c}cbb\bar{b}$	$\bar{a}\bar{b}\bar{c}\bar{c}\bar{b}\bar{b}c\bar{c}\bar{a}cb\bar{b}c\bar{c}$
$(7,22,27)$	$\bar{a}\bar{c}c\bar{b}cb\bar{c}c\bar{a}\bar{c}\bar{c}\bar{c}\bar{c}b$	$\bar{a}\bar{b}b\bar{b}\bar{c}cc\bar{c}\bar{b}\bar{b}cbcc$	$\bar{a}c\bar{b}\bar{b}\bar{b}\bar{c}\bar{c}cb\bar{c}\bar{b}bc\bar{c}$	$\bar{a}bcb\bar{a}\bar{b}cb\bar{c}b\bar{a}\bar{b}c\bar{b}$
$(7,23,26)$	$\bar{a}\bar{c}\bar{b}\bar{c}cccc\bar{c}bc\bar{b}bc$	$\bar{a}c\bar{a}\bar{c}bc\bar{c}\bar{b}\bar{a}bc\bar{c}\bar{c}\bar{c}$	$\bar{a}\bar{b}b\bar{c}c\bar{c}bbb\bar{c}\bar{c}\bar{b}c\bar{b}$	$\bar{a}c\bar{b}\bar{b}\bar{b}\bar{c}\bar{b}b\bar{b}\bar{b}b\bar{c}\bar{a}\bar{b}$
$(8,9,39)$	$\bar{a}bccc\bar{b}ccc\bar{b}\bar{c}\bar{c}\bar{a}c$	$\bar{a}\bar{b}\bar{c}c\bar{c}\bar{c}c\bar{c}\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}c$	$\bar{a}c\bar{a}c\bar{c}b\bar{c}\bar{b}\bar{c}c\bar{c}ccc$	$\bar{a}cc\bar{c}cc\bar{a}\bar{c}ccc\bar{b}c\bar{b}$
$(8,13,35)$	$\bar{a}ccccb\bar{c}\bar{b}\bar{c}\bar{c}\bar{a}b\bar{b}\bar{c}$	$\bar{a}\bar{c}c\bar{c}ccc\bar{a}c\bar{c}b\bar{b}\bar{b}c$	$\bar{a}c\bar{c}\bar{c}c\bar{c}\bar{c}\bar{a}\bar{b}c\bar{b}\bar{c}bc$	$\bar{a}c\bar{b}\bar{c}\bar{a}\bar{c}cbb\bar{c}\bar{c}cc\bar{c}$
$(8,15,33)$	$\bar{a}\bar{c}\bar{c}\bar{b}c\bar{c}c\bar{c}\bar{c}\bar{c}\bar{c}b\bar{b}\bar{c}$	$\bar{a}cccc\bar{c}c\bar{c}\bar{b}cb\bar{c}bc$	$\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}c\bar{a}bbc\bar{c}cb\bar{a}$	$\bar{a}\bar{a}\bar{b}c\bar{c}b\bar{b}\bar{c}\bar{a}cc\bar{b}\bar{b}c$
$(8,17,31)$	$\bar{a}\bar{c}\bar{c}b\bar{c}\bar{c}\bar{b}b\bar{b}\bar{c}\bar{a}\bar{b}\bar{c}\bar{c}$	$\bar{a}c\bar{b}\bar{c}\bar{a}\bar{c}bcc\bar{c}cbb\bar{c}$	$\bar{a}\bar{c}c\bar{b}\bar{c}bbc\bar{a}c\bar{c}\bar{b}cc$	$\bar{a}cb\bar{b}c\bar{c}\bar{a}ccc\bar{c}c\bar{b}\bar{b}$
$(8,21,27)$	$\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}c\bar{b}\bar{c}\bar{c}\bar{b}c$	$\bar{a}c\bar{b}\bar{a}cc\bar{b}c\bar{b}\bar{c}\bar{a}\bar{c}c\bar{b}$	$\bar{a}\bar{b}\bar{c}\bar{b}bb\bar{c}cb\bar{b}ccb\bar{c}$	$\bar{a}bbb\bar{a}\bar{c}cb\bar{c}\bar{b}b\bar{a}cb$
$(9,12,35)$	$\bar{a}ccbc\bar{a}\bar{c}\bar{c}\bar{c}\bar{c}\bar{a}\bar{c}b\bar{b}$	$\bar{a}cb\bar{b}\bar{a}\bar{c}\bar{c}b\bar{c}c\bar{c}c\bar{c}\bar{c}$	$\bar{a}cc\bar{c}\bar{c}c\bar{b}\bar{b}c\bar{a}\bar{c}\bar{b}\bar{c}c$	$\bar{a}\bar{c}\bar{c}c\bar{c}cc\bar{b}\bar{c}\bar{a}cbb\bar{c}$
$(9,14,33)$	$\bar{a}\bar{c}c\bar{b}\bar{b}c\bar{a}cc\bar{b}bcc\bar{c}$	$\bar{a}\bar{c}\bar{b}\bar{a}\bar{c}\bar{c}\bar{c}bcbc\bar{a}\bar{c}\bar{c}$	$\bar{a}\bar{c}\bar{c}b\bar{b}ccc\bar{c}bbcc\bar{a}$	$\bar{a}\bar{a}bc\bar{c}c\bar{c}c\bar{c}bc\bar{b}\bar{c}c$
$(9,15,32)$	$\bar{a}c\bar{b}\bar{a}c\bar{b}\bar{c}ccc\bar{c}\bar{a}\bar{b}c$	$\bar{a}\bar{c}bbcc\bar{a}b\bar{c}b\bar{c}\bar{c}c\bar{c}$	$\bar{a}\bar{c}b\bar{b}\bar{c}\bar{c}c\bar{a}\bar{b}\bar{c}cbcc$	$\bar{a}b\bar{b}\bar{b}\bar{c}c\bar{c}\bar{a}bcccc\bar{c}$
$(9,16,31)$	$\bar{a}cbcccca\bar{a}\bar{b}\bar{b}cc\bar{b}$	$\bar{a}\bar{a}\bar{c}c\bar{c}b\bar{c}cc\bar{b}\bar{c}b\bar{c}b$	$\bar{a}\bar{c}\bar{c}\bar{c}cc\bar{b}\bar{a}bc\bar{c}\bar{b}\bar{c}\bar{b}$	$\bar{a}c\bar{c}\bar{b}\bar{c}\bar{c}\bar{a}cb\bar{c}c\bar{b}bc$
$(9,17,30)$	$\bar{a}c\bar{b}\bar{b}\bar{a}cc\bar{c}\bar{a}\bar{c}b\bar{b}cc$	$\bar{a}b\bar{b}b\bar{c}\bar{b}\bar{a}cbccccc$	$\bar{a}c\bar{c}bb\bar{c}c\bar{c}cc\bar{a}\bar{c}\bar{b}\bar{b}$	$\bar{a}\bar{c}bcbc\bar{c}\bar{c}c\bar{b}\bar{a}bc\bar{c}$
$(9,21,26)$	$\bar{a}\bar{c}\bar{b}c\bar{c}b\bar{b}c\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}c$	$\bar{a}\bar{c}b\bar{c}\bar{b}\bar{b}b\bar{a}b\bar{c}\bar{c}bc\bar{c}$	$\bar{a}bbcc\bar{c}ca\bar{a}ccb\bar{b}\bar{c}$	$\bar{a}\bar{a}\bar{b}bc\bar{c}\bar{b}\bar{a}c\bar{b}\bar{b}\bar{b}\bar{b}b$
$(9,23,24)$	$\bar{a}\bar{b}\bar{b}b\bar{b}\bar{c}\bar{c}\bar{c}cc\bar{c}c\bar{a}c$	$\bar{a}cb\bar{c}bc\bar{b}c\bar{c}ccc\bar{b}\bar{b}$	$\bar{a}\bar{c}\bar{a}b\bar{a}\bar{b}ccbbc\bar{c}\bar{a}\bar{b}$	$\bar{a}\bar{b}\bar{c}b\bar{b}b\bar{a}c\bar{b}\bar{b}cb\bar{b}b$
$(10,11,35)$	$\bar{a}c\bar{a}\bar{c}\bar{c}b\bar{c}b\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}$	$\bar{a}\bar{b}cc\bar{c}\bar{c}\bar{a}\bar{c}c\bar{c}cbc\bar{b}$	$\bar{a}cc\bar{c}c\bar{b}\bar{a}\bar{b}\bar{c}cc\bar{c}\bar{c}b$	$\bar{a}\bar{c}c\bar{b}\bar{c}bac\bar{a}\bar{c}\bar{c}\bar{c}cc$
$(10,13,33)$	$\bar{a}\bar{c}c\bar{b}c\bar{c}\bar{c}\bar{c}\bar{c}b\bar{c}c\bar{a}\bar{c}$	$\bar{a}\bar{b}\bar{c}b\bar{c}cc\bar{c}\bar{c}\bar{b}\bar{c}\bar{b}\bar{c}\bar{c}$	$\bar{a}\bar{c}abc\bar{b}cc\bar{a}\bar{c}c\bar{b}\bar{a}\bar{b}$	$\bar{a}b\bar{a}\bar{c}\bar{c}\bar{b}\bar{c}c\bar{a}\bar{b}\bar{c}cc\bar{c}$
$(10,15,31)$	$\bar{a}\bar{c}\bar{a}c\bar{c}c\bar{b}bcccc\bar{c}\bar{c}$	$\bar{a}\bar{c}c\bar{b}\bar{b}\bar{c}\bar{c}\bar{b}c\bar{c}cc\bar{a}c$	$\bar{a}\bar{a}\bar{c}b\bar{c}\bar{c}bcbc\bar{c}\bar{b}c\bar{a}$	$\bar{a}b\bar{a}\bar{b}bcc\bar{b}\bar{a}c\bar{c}bbc$
$(10,17,29)$	$\bar{a}\bar{b}\bar{c}cb\bar{a}cccc\bar{c}\bar{b}\bar{a}c$	$\bar{a}\bar{c}c\bar{c}\bar{c}ccc\bar{b}bbcb\bar{a}$	$\bar{a}\bar{c}\bar{a}\bar{b}\bar{c}\bar{c}\bar{b}cc\bar{a}\bar{b}\bar{c}c\bar{c}$	$\bar{a}\bar{a}b\bar{b}\bar{b}cb\bar{c}c\bar{b}c\bar{b}c\bar{b}$
$(10,21,25)$	$\bar{b}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}cc\bar{b}b\bar{b}c\bar{c}c$	$\bar{a}c\bar{b}\bar{a}bb\bar{c}bcba\bar{c}\bar{c}\bar{a}$	$\bar{a}\bar{a}\bar{b}b\bar{a}\bar{c}b\bar{c}\bar{b}\bar{c}c\bar{a}\bar{c}\bar{b}$	$\bar{a}\bar{c}\bar{b}c\bar{b}\bar{c}c\bar{a}bb\bar{c}\bar{b}\bar{c}b$
$(11,12,33)$	$\bar{a}c\bar{a}bcc\bar{c}b\bar{c}bc\bar{c}\bar{a}c$	$\bar{a}\bar{c}\bar{c}c\bar{a}\bar{b}cccc\bar{a}b\bar{c}\bar{b}$	$\bar{a}\bar{c}\bar{c}\bar{b}\bar{a}\bar{c}\bar{c}\bar{c}\bar{c}bcb\bar{a}\bar{c}$	$\bar{a}\bar{b}\bar{a}\bar{c}c\bar{c}\bar{c}\bar{c}cb\bar{c}\bar{b}c\bar{c}$
$(11,13,32)$	$\bar{a}\bar{c}b\bar{b}\bar{c}c\bar{a}\bar{b}cccc\bar{c}\bar{a}$	$\bar{a}\bar{c}\bar{b}cb\bar{c}b\bar{c}c\bar{b}\bar{c}ccc$	$\bar{a}\bar{a}cb\bar{a}c\bar{c}abccc\bar{c}\bar{b}$	$\bar{a}\bar{c}\bar{a}c\bar{c}\bar{b}cc\bar{b}b\bar{c}c\bar{a}c$
$(11,15,30)$	$\bar{a}\bar{c}\bar{b}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}cb\bar{c}\bar{c}$	$\bar{a}\bar{b}b\bar{c}b\bar{b}b\bar{c}\bar{c}\bar{c}\bar{c}c\bar{c}\bar{c}$	$\bar{a}c\bar{c}\bar{c}\bar{a}bb\bar{c}cacc\bar{b}\bar{a}$	$\bar{a}\bar{a}\bar{a}c\bar{c}c\bar{b}\bar{c}\bar{c}b\bar{b}a\bar{a}\bar{b}$
$(11,16,29)$	$\bar{a}\bar{c}\bar{c}\bar{a}\bar{c}\bar{c}c\bar{c}c\bar{a}cc\bar{b}\bar{c}$	$\bar{a}\bar{c}\bar{c}\bar{a}ccbcc\bar{c}c\bar{a}bb$	$\bar{a}cc\bar{c}b\bar{b}cc\bar{b}\bar{b}bc\bar{c}\bar{b}$	$\bar{a}\bar{a}c\bar{b}b\bar{b}\bar{c}bbba\bar{a}\bar{c}c$
$(11,22,23)$	$\bar{a}\bar{c}c\bar{a}c\bar{b}\bar{c}cb\bar{c}\bar{c}\bar{c}\bar{c}\bar{a}$	$\bar{a}c\bar{b}b\bar{c}cc\bar{c}\bar{a}b\bar{b}\bar{b}\bar{b}\bar{c}$	$\bar{a}\bar{c}\bar{a}\bar{b}\bar{c}b\bar{a}c\bar{b}cb\bar{b}\bar{b}\bar{b}$	$\bar{a}\bar{a}bb\bar{b}\bar{c}b\bar{b}c\bar{b}\bar{c}b\bar{a}\bar{c}$
$(12,13,31)$	$\bar{a}\bar{b}\bar{c}\bar{b}\bar{b}\bar{c}ca\bar{a}cccc\bar{c}$	$\bar{a}b\bar{c}\bar{b}\bar{a}c\bar{c}cb\bar{c}\bar{a}ccc$	$\bar{a}\bar{a}c\bar{c}\bar{a}\bar{c}cccb\bar{c}c\bar{b}\bar{c}$	$\bar{a}\bar{b}c\bar{b}cb\bar{b}a\bar{c}b\bar{a}\bar{c}\bar{c}c$
$(12,15,29)$	$\bar{a}\bar{c}\bar{c}\bar{b}c\bar{b}\bar{c}\bar{c}b\bar{c}\bar{c}\bar{c}\bar{c}c$	$\bar{a}c\bar{c}\bar{c}\bar{b}cbacc\bar{a}cc\bar{c}$	$\bar{a}\bar{b}bcc\bar{b}\bar{b}c\bar{c}\bar{c}ba\bar{a}\bar{b}$	$\bar{a}\bar{a}\bar{a}cc\bar{c}\bar{a}a\bar{b}\bar{b}\bar{c}bbc$
$(12,17,27)$	$\bar{a}\bar{a}\bar{c}c\bar{a}ac\bar{c}\bar{c}\bar{c}\bar{c}\bar{b}\bar{a}\bar{c}$	$\bar{a}c\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}\bar{c}bc\bar{c}\bar{a}cb$	$\bar{a}c\bar{b}\bar{b}cb\bar{a}cbb\bar{c}b\bar{b}c$	$\bar{a}\bar{c}\bar{a}\bar{b}b\bar{a}c\bar{b}\bar{c}\bar{b}\bar{b}cb\bar{b}$
$(13,16,27)$	$\bar{a}\bar{b}\bar{c}c\bar{b}b\bar{c}\bar{a}\bar{b}\bar{c}\bar{c}\bar{c}\bar{c}c$	$\bar{a}b\bar{a}bc\bar{a}ccc\bar{c}\bar{c}cbc$	$\bar{a}\bar{c}cb\bar{b}\bar{b}b\bar{c}b\bar{c}\bar{a}bb\bar{c}$	$\bar{a}\bar{a}\bar{b}\bar{b}\bar{a}a\bar{c}c\bar{c}acc\bar{a}\bar{c}$
$(13,19,24)$	$\bar{a}\bar{c}c\bar{c}\bar{b}c\bar{b}\bar{c}\bar{c}cccc\bar{a}$	$\bar{a}\bar{a}\bar{c}\bar{a}abb\bar{c}\bar{b}\bar{c}a\bar{c}cc$	$\bar{a}\bar{a}cc\bar{c}cbaa\bar{a}bb\bar{b}b$	$\bar{a}b\bar{b}cbc\bar{b}b\bar{c}bc\bar{b}\bar{b}\bar{b}$
$(13,20,23)$	$\bar{a}\bar{a}\bar{b}\bar{a}\bar{c}c\bar{b}\bar{b}\bar{c}cccc\bar{b}$	$\bar{a}cccb\bar{a}\bar{c}\bar{b}\bar{c}c\bar{b}\bar{c}b\bar{b}$	$\bar{a}b\bar{a}c\bar{c}ccb\bar{b}bb\bar{c}\bar{a}c$	$\bar{a}\bar{b}\bar{b}\bar{b}bab\bar{a}ab\bar{c}\bar{a}c\bar{c}$
$(13,21,22)$	$\bar{a}\bar{b}\bar{c}\bar{b}\bar{c}\bar{c}\bar{c}cc\bar{b}bc\bar{c}c$	$\bar{a}\bar{a}\bar{b}ccbb\bar{a}c\bar{c}\bar{a}a\bar{a}b$	$\bar{a}\bar{a}bb\bar{b}\bar{a}\bar{b}bb\bar{c}\bar{c}bbc$	$\bar{a}bb\bar{b}b\bar{c}a\bar{c}b\bar{a}c\bar{c}\bar{c}c$
$(15,17,24)$	$\bar{a}\bar{b}\bar{c}c\bar{b}\bar{b}ba\bar{a}\bar{c}cccc$	$\bar{a}cc\bar{a}c\bar{b}c\bar{c}\bar{c}\bar{b}c\bar{a}b\bar{c}$	$\bar{a}\bar{a}bb\bar{b}\bar{c}c\bar{a}\bar{c}\bar{b}\bar{a}\bar{b}bc$	$\bar{a}\bar{a}\bar{c}bbcb\bar{c}ca\bar{b}a\bar{a}\bar{c}$
$(15,19,22)$	$\bar{a}\bar{a}\bar{c}c\bar{b}b\bar{b}\bar{b}\bar{a}c\bar{c}\bar{c}\bar{c}\bar{c}$	$\bar{a}bb\bar{a}\bar{c}c\bar{a}\bar{c}\bar{b}\bar{b}b\bar{a}cc$	$\bar{a}\bar{b}\bar{c}\bar{b}\bar{c}\bar{b}b\bar{b}\bar{b}ca\bar{a}c\bar{b}$	$\bar{a}c\bar{a}\bar{c}c\bar{a}cabb\bar{c}\bar{a}\bar{b}c$
$(16,17,23)$	$\bar{a}\bar{a}\bar{c}c\bar{a}a\bar{b}bbcccc\bar{c}$	$\bar{a}\bar{a}abb\bar{c}c\bar{c}\bar{a}c\bar{c}\bar{c}\bar{c}\bar{a}$	$\bar{a}\bar{b}\bar{b}cc\bar{a}bc\bar{c}\bar{b}bc\bar{a}b$	$\bar{a}\bar{c}\bar{a}c\bar{b}b\bar{b}ca\bar{a}\bar{b}\bar{c}bb$
$(17,19,20)$	$\bar{a}\bar{c}\bar{a}\bar{c}\bar{b}ccc\bar{a}\bar{c}cc\bar{c}c$	$\bar{a}\bar{b}c\bar{a}bcc\bar{b}\bar{b}\bar{c}\bar{b}b\bar{a}\bar{b}$	$\bar{a}\bar{a}\bar{b}\bar{c}bb\bar{c}b\bar{b}bc\bar{a}bb$	$\bar{a}\bar{a}b\bar{a}c\bar{c}a\bar{a}cabа\bar{a}\bar{b}$