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Number of Metric Bases up to Automorphism

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Let G be a connected graph. A subset S of vertices of G is called a *resolving set* if for every pair of vertices $g_1, g_2 \in V(G)$, there exists $s \in S$ with $\mathrm{d}(g_1, s) \neq \mathrm{d}(g_2, s)$. A resolving set of minimum cardinality is called a *metric basis* for G, and the cardinality of such a set is called the *metric dimension* $\dim(G)$.

Let S be a metric basis of G. In certain applications of metric dimension, such as robot navigation, S plays a crucial role, since it uniquely determines the position of each vertex in the graph. Now, suppose that specifying the position of each vertex using S is not allowed. Therefore, a new metric basis S' is required. However, finding a metric basis is a difficult task as it is proven to be NP-hard. Therefore, the process of finding S' is time-consuming. Here, we reduce this calculation by exploiting the symmetries (automorphisms) of the graph. It is demonstrated that it is not necessary to find all metric bases, as a certain number of them suffices, and the remaining metric bases can be derived from them. The following observation is of pivotal importance to the defined concept.

Observation 0.1 Let G be a connected graph with a resolving set S. Then for any $f \in \mathrm{Aut}(G)$, $f(S)$ is a resolving set.

Let $\mathfrak{S}(G)$ be the set of all metric bases of G. Let $\approx$ be a relation on $\mathfrak{S}(G)$ defined by $S_1 \approx S_2$ if and only if there exists $f \in \mathrm{Aut}(G)$ such that $f(S_1) = S_2$, for any $S_1, S_2 \in \mathfrak{S}(G)$. Clearly, $\approx$ is an equivalence relation on $\mathfrak{S}(G)$. A *resolving class* of a resolving set S, is $[S] = \{f(S) \mid f \in \mathrm{Aut}(G)\}$. Then, for any $S_1, S_2 \in [S]$, $S_1 \approx S_2$; and $S_1 \not\approx S_2$ if and only if $[S_1] \neq [S_2]$. Let $\mathfrak{B}(G)$ be the maximal subset of $\mathfrak{S}(G)$ such that for any two sets S_1 and S_2 in $\mathfrak{B}(G)$, we have $[S_1] \neq [S_2]$. The set $\mathfrak{B}(G)$ contains exactly one representative of each resolving class. We say that $\mathfrak{B}(G)$ is a *resolving basis* of G. Indeed, $\mathfrak{B}(G)$ represents a generator for all resolving sets, which may not necessarily be the smallest possible size. By combining $\mathfrak{B}(G)$ with the automorphism group of G, it is possible to generate all resolving sets of G. In this talk, we examine some results on $\mathfrak{B}(G)$ and present a delineation of open problems and potential areas for future research.

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