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Erdős-Ko-Rado Type Results for Intersecting Families of Subgraphs of Complete Graphs

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The Erdős-Ko-Rado (EKR) theorem is a fundamental result in extremal set theory. It was proved by Erdős, Ko, and Rado in 1938 and later published in 1961. It states that if n and k are positive integers satisfying $n \geq 2k$, then the maximum size of an intersecting family of k-element subsets of the set $\{1, 2, \ldots, n\}$ is $\binom{n-1}{k-1}$. Furthermore, when $n > 2k$, this bound is attained if and only if the family consists of all k-subsets containing a fixed element of $[n]$. Since the original proof, numerous generalizations and alternative proofs of the theorem have been developed. For example, in 1977, Deza and Frankl showed that the maximum size of an intersecting family of permutations on $[n]$ is $(n-1)!$. Also, in 2005, Meagher and Moura showed that in the complete graph on an even number of vertices, the largest size of an intersecting family of perfect matchings is precisely the family of all the perfect matchings that share one specific edge. In 2017, Godsil and Meagher presented an algebraic proof of this result. In 2013, Kamat and Misra extended this result to the family of k-matchings.

In this context, the following natural questions arise.

A family $\mathcal{F}$ of subgraphs of the complete graph K_n is said to be intersecting if any two subgraphs in $\mathcal{F}$ share at least one common edge.

- What is the maximum possible size of an intersecting family of k-cycles in K_n, and what is the structure of the intersecting families that attain this maximum?
- Let $\mathcal{H}$ denote the family of all subgraphs of K_n that are isomorphic to a given graph H. What is the maximum size of an intersecting subfamily of $\mathcal{H}$, and what is the structure of the intersecting families that attain this maximum?

In this talk, we introduce a composition lemma for ErdősKoRado families and use it to answer the above questions. In particular, we show that if n is sufficiently large relative to k, then the largest intersecting families of k-cycles in the complete graph K_n are exactly those where all k-cycles share a common edge. We also show that the same result holds for the largest intersecting subfamily of $\mathcal{H}$, where $\mathcal{H}$ is the family of all subgraphs of K_n that are isomorphic to a given graph H.

This is a joint work with Javad B. Ebrahimi.