On Weakly Connected Domination in Graphs

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Abstract

Let $G = (V, E)$ be a graph. A subset S of V is called a *dominating set* of G if every vertex in $V \backslash S$ is adjacent to a vertex in S. The *domination number* of G, denoted by $\gamma(G)$, is defined as

$$\gamma(G) = \min\{|S| : S \text{ is a dominating set in } G\}.$$

A dominating set S of G is called a *connected dominating set* if the induced subgraph $[S]$ of G is connected. The *connected domination number* of G, denoted by $\gamma_c(G)$, is defined as

$$\gamma_c(G) = \min\{|S| : S \text{ is a connected dominating set in } G\}.$$

The subgraph $[S]_w$ *weakly* induced by S in G is defined as the subgraph $(N[S], E \cap (S \times N[S]))$ in G. A dominating set S of G is called a *weakly connected dominating set* of G if the subgraph $[S]_w$ is connected. The *weakly connected domination number* of G, denoted by $\gamma_w(G)$, is defined as

$$\gamma_w(G) = \min\{|S| : S \text{ is a weakly connected dominating set in } G\}.$$

It follows from definition that $\gamma(G) \leq \gamma_w(G) \leq \gamma_c(G)$. In this talk, we give a brief survey on some fundamental results about $\gamma_w(G)$, which include the relationships among the above three parameters.